The Optimal Design of NOME-type Regulation in Greece*

Report prepared fro the Greek Regulatory Authority for Energy†

Costas Courcoubetis[1], Yiannis Katsoulakos[1], and George Stamoulis[1]

[1]Athens University of Economics and Business

Final Report v.1 - July 2013

Chapter 2: The Implications of NOME in a uniform price auction-operated wholesale electricity market

1 Introduction

The objective of this Chapter of the report is the analysis of the properties of the uniform price auction for the Greek electricity market taking into account the effects of the NOME regulation. The basic goal is to compute the price equilibria that will result by the competition of DEH and the three smaller competitors with and without using NOME, and analyze the effects on these prices of the various parameters of the NOME regulation, i.e., of α, ϕ, and the price p_a. Then, reasoning backwards, one can use these results to provide guidelines for the choice of these parameters by the regulator (these are summarized in Chapter 5 of the report) .

To our knowledge there is no existing analysis of the uniform price auction that can be applied in our case and help us in calculating the different resulting price equilibria as a function of the demand and the system parameters. To solve this problem we had to conduct our own theoretical analysis of the auction for the general case, and prove some important structural properties that allow us to simplify the search for Nash equilibria (NEs). In particular we have characterized the possible NEs as a function of the system parameters and supplied some necessary and sufficient conditions that are effectively computable. This allowed us to derive computational procedures for finding these equilibria which we implemented in a specialized auction analysis tool. This theoretical part of our contribution and the description of the computational procedures for computing Nash equilibria is in the Appendix of this Chapter. A summary of our analysis results of the Greek wholesale market is as follows.

1. **Market analysis**. In Section 2 of this Chapter we show the price equilibria with and without NOME for as a function of the values of the demand (which we assume inelastic to prices). We then compute the profits of the NOME providers and analyze their incentives to make use of the NOME regulation. We also use our auction analysis tool to analyze the sensitivity of the price equilibria w.r.t. key parameters such as the values of α, p_a, the symmetry of the NOME lignite allocation, the maximum price P_{max} that one is allowed to use, and the composition ϕ of the NOME mixture. We observe

*We are grateful to Claude Crampes and Patrick Rey for useful discussions concerning the analysis contained in this report. Of course, all errors, ambiguities and omissions remain solely our responsibility.

†We are grateful to Eleni Metsiou and Angeliki Anastopoulou for research assistance during the preparation of this report.

that benefits of the NOME regulation in generating lower price equilibria than in the case without such regulation. We also observe the beneficial effects of using low values of ϕ in the range of 0.5-0.6 in extending the range of low price equilibria, but at the cost of increasing the value of this low price (the cost of the NOME mixture).

2. **Forward contracts**. In Section 3 we analyze the effects of forward contracts to the price equilibria of the uniform price auction. As expected, when the providers (and in particular the incumbent) have large amounts of froward contracts and become buyers from the pool, their incentives for price setting at higher prices are reduced. This results in lower price equilibria for the same demand values. We again analyze the incentives of the small providers to adopt NOME and perform a sensitivity analysis w.r.t. the allocation of the forward contracts to the various providers.

3. **Adding practical constraints**. In this section (Section 4) we attempt to model the existence of costs in turning on and off lignite facilities. The general problem is extremely hard to analyze exactly. To make it tractable we use a simple heuristic under which DEH is not price setting its lignite production at P_{max}. This is justified because for medium to moderately high demand, the amount of lignite capacity sold in the unconstrained equilibrium is small, implying that DEH will have to turn most of its lignite production off or sell it below cost. We analyze the resulting price equilibria under NOME and see again the sensitivity under various system parameters. We must stress that this part of our report is based on a heuristic that could be debated, and hence should be considered only as providing a lower bound on prices for values of demand that are not at the high range.

4. **Linking the wholesale prices to strategies in the NOME auction**. In Section 5 we discuss the incentives of providers to act strategically in the NOME lignite allocation auction, when they anticipate the profits from the whole sale market in the future, where these prices are determined by the uniform auction. This combination of the NOME allocation auction and the subsequent operation of the whole sale market using the pool is modeled as a two stage game. In the first stage (the NOME allocation game) each provider buys the amount of NOME he will use to bid in the uniform price auction in the second stage. Based on the profits in the second stage he decides on the amount to bid in the first stage. We show under certain assumptions that the equilibrium strategy of the first stage is to buy the maximum possible quantity of lignite to be used at the second stage. This property holds for all value of demand.

5. **A preliminary analysis of the retail market in the short run**. In Section 6 we analyze the prices that will take place in the wholesale market and the retail market when full competition at the latter market is not established and NOME providers can only sell their NOME production in retail while DEH is supporting the rest of the market. This condition is restrictive in general but it might hold in the aftermath of the NOME regulation when competition in the retail market is not fully established. We observe that DEH has in this case incentives to keep wholesale price lower than before, since it is a net buyer from the pool.

We proceed now with the detailed presentation of our results.

2 Market analysis

In this section we use the computational procedures described in the Appendix to analyze the deregulated Greek wholesale market assuming that prices are determined by using a mandatory pool operating under a uniform price auction.

We study using numerical analysis the prices that will take place at all the possible Nash equilibria of the auction when the competing providers have the sizes and power generation capabilities of the Greek electricity market. Our goal is to show the effects of NOME on the equilibrium prices for a range of demand parameters, and also study the sensitivity of these results to key parameters of the NOME regulation. We must stress that our analysis is attempting to capture the 'first-order' effects of competition in this market. It is not designed to be 100% realistic, and it is simplified as much as possible in order to be analytically tractable. For this reason we assume an oligopoly of four providers, with fixed marginal costs of electricity production that depend on the combustible used.

Our bidding model in the auction assumes the highest possible flexibility from the side of the competing providers to act strategically and price their products in the auction. we don't consider the effects of 'turning-off-and-on' costs to the above strategies, i.e., the costs of turning off and on their generation plants in the case that for some time period a power plant exhibiting the above costs due to a high bid was not selected to participate in power generation by the auctioneer. The impact of such costs to strategies is discussed later in Section 4.1, and complicates considerably the analysis since the decision of a player on how to bid at a given time depends on the current state of its power plants and also on the expectation of the demand at the future times. This explains the behavior of strategic under-biding, i.e., bidding below marginal cost or avoiding price setting at very high prices (which will result in lower quantities being sold by the bidder but at very high profit).

Hence in this section we consider the basic auction model of the Appendix where actions of players are myopic and based on their current variable cost information and do not take into account their effect in the future operation (and hence cost) of the system. For instance, we assume that the cost of lignite is its variable cost of providing power under continuous operation, and we don't take into account the extra potential costs of a strategy that decides on turning some lignite plants off to create artificial scarcity and raise prices temporarily.

In principle one could add these extra cost in an expected sense in the model, for instance, by raising the average cost of a lignite facility if in the long term this facility due to strategic bidding is being turned off and on with some known frequency. Hence, one would expect that our model can capture the study state operation of the market, where the costs used to make decisions are adjusted to reflect the actual average operating costs of the plants under the given strategies. In our present analysis we are not able to predict these costs and hence we use the available average costs that correspond to the continuous operation of the plants. One can argue that this analysis corresponds to a 'worse case' analysis of the market since strategic price setting at high prices above marginal costs implies a smaller quantity sold by the strategic bidder and hence a highest occurrence of turning off-and-on costs. These extra costs will discourage such strategic behavior that contributes to high prices. Another way to see this is that by incorporating these costs to the average production costs we raise the marginal costs. This reduces the benefit from strategic price setting[1].

[1]We use the term 'price setting' to refer to the strategic behavior that a player in the auction prefers to raise his prices and sell a small quantity at a higher price than remain truthful and sell a larger quantity at the competitive market price (prices reflect actual costs). The analysis of such strategies is in the Appendix.

To summarize the characteristics of the key energy providers that are expected to participate in the auction we make the following assumptions[2]:

- The incumbent provider is DEH with the following range of products (according to the energy source used): Lignite with $K^0_{l,I} = 120$ and $c_{l,I} = 40$ (using the appropriate units, the superscript 0 is used to denote the initial total lignite capability of the incumbent), gas with $K_{g,I} = 20$ and $c_{g,I} = 80$, and petrol with $K_{h,I} = 40$ and $c_{h,I} = 100$.
- Competition is provided by three competing providers. For simplicity we consider them symmetric in their production capabilities. Each provider E has initially (with no use of NOME contracts) only gas with $K_{g,E} = 20$ and $c_{g,E} = 80$. Let $c_g = c_{g,E} = c_{g,I}$.
- Demand is in the range 0-240.

Unless otherwise stated, we assume that there is a maximum price P_{max} in the auction imposed by the regulator.

If NOME is used, then a fraction α of the lignite capability of DEH is assigned to the three competitors at a known unit price p_a. In order to resell it in the wholesale market (i.e., in the auction) the NOME requirement is that they must mix it with a some gas production. More specifically, each unit of energy produced using NOME contracts must be generated by a fraction ϕ of energy based on lignite (obtained from DEH) and $(1 - \phi)$ of energy produced using their own gas production. This implies that the marginal cost of energy generated by the 'virtual' NOME facility using the above mixture is $c_N = \phi c_{g,E} + (1 - \phi)p_a$. For instance, if $\phi = 0.8$, then $c_N = 52$.

We assume for simplicity that the amount of lignite allocated using NOME to a competing provider is below the maximum amount that would require all the gas capability of the provider to generate the appropriate mix, i.e., less than $\phi/(1 - \phi)K_{g,E}$.

Explanation of the plots

Our plots that analyze the auction have on the x-axis the value of the demand and on the y-axis the value of the price. In each figure we display simultaneously many plots, that differ in color and line characteristics. For each different potential equilibrium price $p \in \{c_{l,I}, c_N, c_g, c_{h,I}, P_{max}\} = \{40, 52, 80, 100, 150\}$ we have a plot that consists of the indicator function taking the value one for the values of the demand θ for which there is a non-truth telling NE with equilibrium price p, multiplied by p. Note that our theory precludes the existence of an equilibrium at $c_{l,I}$ since only DEH competes when θ is below $K_{l,l}$, and hence it will raise its price to the next MC that is c_N or c_g. Hence there may be at most four plots, corresponding to $p \in \{c_N, c_g, c_{h,I}, P_{max}\} = \{52, 80, 100, 150\}$. Each such plot has a different color and corresponds to $pf^2_p(\theta)$ for the given p. If $f^2_p(\theta) = 0$ for all θ, i.e., there is no range of demand values for which p is an equilibrium price for some non-truth telling NE, then the corresponding plot is empty (does not exist).

In addition to the above plots, we plot using a dotted line the value of the truth telling market price, when all providers bid truthfully. From our analysis we know that such a price might be also an NE price, iff for that value of the demand there is no non-truth telling NE. For each given demand, if there is no equilibrium where the price is set above its MC by

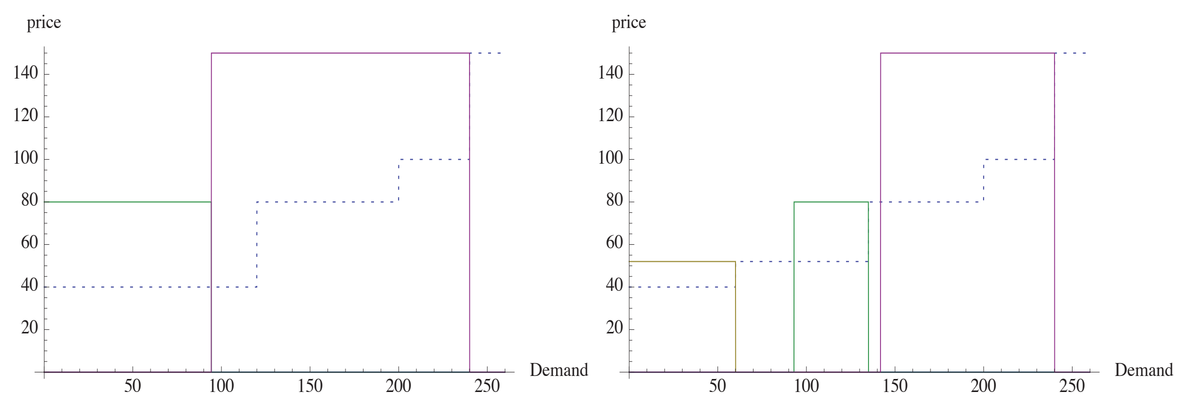
[2] The quantities we use are scaled versions of the actual parameters of the Greek market using data provided by RAE. In our analysis we show also the unscaled results.

some provider, then the only possible NE is the truth telling one. In a figure plotting the equilibrium prices, if for some demand value the only price plot value that corresponds is the truth telling one, then this is also the NE price. If there is some overlapping plot for some p in the candidate set of prices, then this is an equilibrium price, and truth telling is no more an equilibrium strategy.

2.1 Results

2.1.1 With and without NOME

We analyze and compare the cases where the market operates without NOME (i.e., $\alpha = 0$) and the case where $\alpha = 0.50$, i.e., half of DEH's lignite production is shared among the three competing gas providers. We also assume that these providers are symmetric and obtain an equal share of the NOME lignite production. In Figure 1 (a) we show the price equilibria when no NOME is used, and in Figure 1 (b) we show these equilibria when $\alpha = 0.50$.

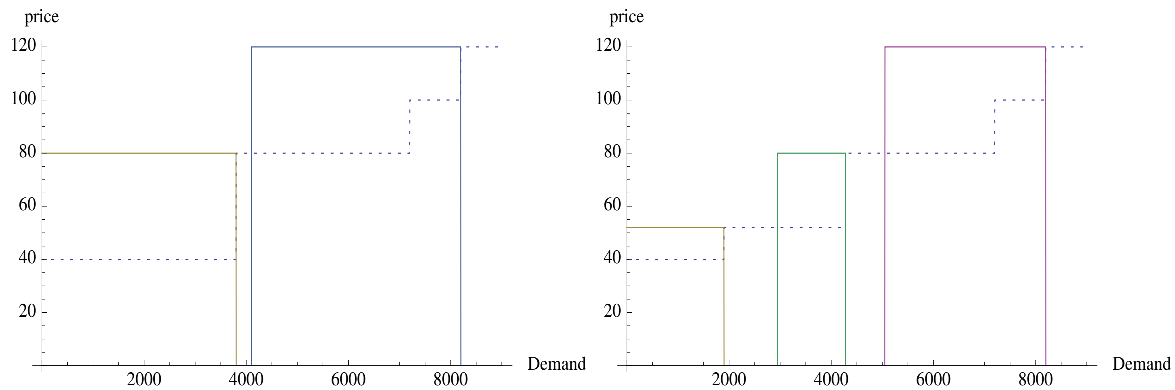


(a) Market prices without NOME ($\alpha = 0$)

(b) Market prices with NOME for $\alpha = 0.50$ and $\phi = 0.80$

Figure 1: (a) We observe that for low values of the demand the equilibrium price is the price of gas 80, and for medium to high $\theta > 100$ the price rises to $P_{max} = 150$. The prices are higher than actual MC due to DEHs strategic price setting.
(b) We observe a drastic improvement of the prices compared to the case of $\alpha = 0$. For demand values θ below 100, prices fall to the cost of the NOME mixture. For medium demand, price rise to the cost of gas, and only for high values of demand above 140 prices rise to P_{max}.

Using forecasted (for 2104) market data we show the results in Figure 2. Here we assume for DEH that it has capacity 3800 for lignite, 1200 for gas and 1000 for petrol. The competitors have a total gas capacity of 2200 shared equally.



(a) Figure 1 (a) with actual market values

(b) Figure 1 (b) with actual market values

Figure 2: Without NOME and with NOME for actual market values and $\alpha = 0.50, \phi = 0.8$.

In Figure 1 (a) price are above MC due to strategic bidding by DEH. At $\theta = 80$ DEH bids its its lignite production at the price of gas 80, and ends up selling all 80 units at that price while its competitors bid truthfully their gas at the same price 80, but don't sell anything because DEH has a lower actual MC and is preferred but he rules of the auction. At $\theta = 110$ it bids its lignite production at P_{max} 150, and ends up selling 50 units at that price while its competitors bid truthfully and sell their total gas production of 60 units at the same high price. DEH makes a profit of $50 \times (150 - 40) = 5500$ by selling only 50 units of lignite at the high price instead to be truthful and sell all its production of 120 at actual cost making 0 profit.

There is clear improvement in the market prices we obtain (i.e., lower prices) when NOME is used for low to medium demand values. In the case of $\alpha = 0$ prices are at P_{max} for values of the demand θ above 100, whereas with $\alpha = 0.50$ P_{max} appears when θ is above 140. Similarly, when $\alpha = 0$ for low demand we obtain a market price equal to the cost of gas, whereas with NOME this price drops to the price of the NOME mixture.

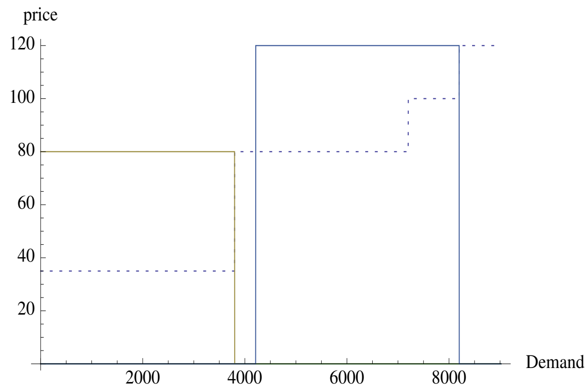
More detailed analysis explains also who is the price setter in Figure 1 (b) for different value of the demand. When demand is in the lignite range, then only DEH is the price setter. The rest of the equilibria with prices at c_g and P_{max} can be due to any of the participants setting the high price. When such an equilibrium starts occurring, DEH is always the price setter. Then as the value of the demand increases, the rest of the providers become also candidates for price setting.

Lets examine this in the setup of Figure 1 (b), in the range of $142 \leq \theta \leq 240$ where we get an a equilibrium at P_{max}. For $\theta = 145$, DEH is the price setter, and the market price is determined by bidding its lignite capability of 60 units at the price of $P_{max} = 150$. Here DEH is selling only 25 units of lignite capacity at price 150. The rest of the providers sell all their capacity at 150. For demand slightly below 142, truthful bidding is the NE. When demand rises above 180, then the price setting bid of DEH is the one of its gas production, which is priced at 150 while lignite now is priced at its marginal cost. For example, if $\theta = 181$, price is set at 150 by selling a single unit of gas at 150. A similar phenomenon occurs when demand crosses 200, in which case price is set to 150 by the petrol production of DEH, whereas all the other bids are truthful.

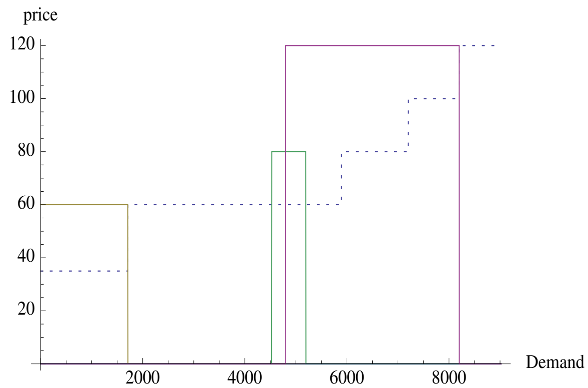
Consider now the case that $\theta > 215$ (say $\theta = 216$). Now all producers are candidate for price setting by bidding non truthfully. Whoever from the three competitors of DEH wants to set the high price, he will sell a quantity of NOME mixture in excess of 15 units (say 16 units if $\theta = 16$) sold at price 150. This capacity is the left-over demand if the rest of the providers are truth telling. Such a provider finds it more profitable to set the high price and sell less than become truth telling and sell more at a lower price. When demand crosses the value of 225, then the price setting bids of the DEH competitors are the ones that correspond to their gas production. The rest of the bids are truthful.

The above analysis shows the complexity of the various equilibria and their justification.

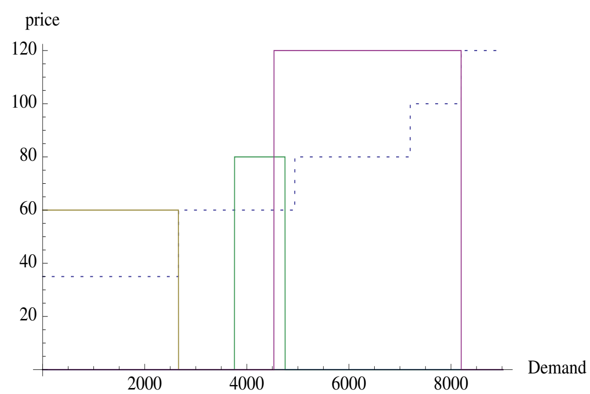
In Figure 3 we show the results of applying NOME when the value of ϕ is much lower than before, and equal to 0.5.



(a) Without NOME



(b) With NOME, $\phi = 0.5, \alpha = 0.55$



(c) With NOME, $\phi = 0.5, \alpha = 0.30$

Figure 3: Without NOME and with NOME for actual market values and $\alpha = 0.55, 0.30$ and $\phi = 0.5$, $c_l = 35, p_a = 40$. The key observation compared to Figure 2 is that the range of low price equilibria at the cost (=60) of the NOME mixture extends further in the medium demand range, and the range where the equilibria are at the price of gas (=80) shrink. Here there is a stronger gain by adopting NOME when ϕ has values less than 0.7. We also observe that for the given value of $\phi = 0.5$, when α changes from 0.55 to 0.30, the equilibrium price for medium demand ($\theta = 4200$) switches from the cost of NOME 60 to the cost of gas 80. Hence the NOME providers benefit by selling less NOME product at medium demand since they can sell it at a profit. For high demand ($\theta = 5200$) the price remains always P_{max}.

2.1.2 Incentives of providers to adopt NOME

So far we discussed how NOME affects the market price. A key issue is whether the providers are better of with NOME or without it. To answer this question we need to compare their profits in the two cases considering the market analysis of Figures 1 (a), (b). In the first, the providers operate without using the NOME capability, while in the second they have exhausted fully their NOME lignite allowance of $\alpha/3$ times the total lignite production of DEH, for $\alpha = 0.50$. The key issue is to analyze their profits in each of these cases. It is not clear that under NOME these profits are always larger since for medium demand the market price is lower under NOME. Hence one has to consider the amounts sold in order to obtain an accurate estimate of the profits.

We plot the profits of the small providers from the auction as a function of the demand in Figure 4.

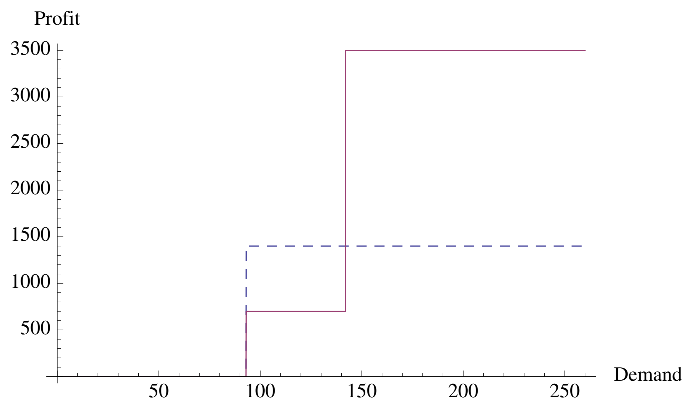


Figure 4: Profits from the auction for a small provider. The dashed line corresponds to the case without NOME and the normal line to the case with NOME.

We will consider three distinct cases according to demand. Note that when $\alpha = 0.50$, since $\phi = 0.8$, the total quantity of the NOME mixture per competitor is $\alpha/3 \times 120/\phi = 25$.

1. Low demand. Consider the case of $\theta = 50$. Here in both cases of $\alpha = 0$ and $\alpha = 0.50$ DEH is setting the equilibrium price and satisfies all the demand by selling its energy from its lignite production. Competitors are selling 0 quantity, and hence make profit 0. When demand increases above 60, then they continue to make profit 0 (in both cases) since with $\alpha = 0$ they sell 0, and with $\alpha = 0.50$ they sell part of their NOME mixture at cost c_N.

2. Medium demand. Take the case of $\theta = 100$. When $\alpha = 0$, DEH is the price setter bidding its lignite production at 150. The competitors are truth telling and sell all their gas production making a total of $3 \times 20 = 60$, while DEH fills the rest of the demand $100 - 60 = 40$ from its lignite production. Each competitor makes profit $20 \times (150 - 80) = 1400$ and DEH makes $40 \times (150 - 40) = 4400$. Observe that DEH is better off seeing this high price than being truth telling and selling 100 units of lignite at cost (this makes zero profit for DEH).

When $\alpha = 0.50$ DEH is again the price setter bidding its lignite production at the price of gas 80. In this case a competitor is truth telling and sells all his NOME production of 25 units at price 80, making a profit of $25 \times (150 - 52) = 700$. DEH sells the rest of $100 - 3 \times 25 = 25$ at price 80 making a profit $25 \times (150 - 40) = 2750$. Observe that this is more than if DEH was truth telling, in which case it would sell all its lignite production at the NOME mixture cost 52 (since in this case the market price would be determined by the truthful bids for the NOME mixture of the competitors), i.e., selling all its lignite at price 52 making profit $60 \times (52 - 40) = 720$.

Summarizing, a competitor makes a profit of 1400 when $\alpha = 0$, and a profit of 700 when $\alpha = 0.50$. The difference in profits of these two cases is 700, and it clear that a competitor is better off when $\alpha = 0$.

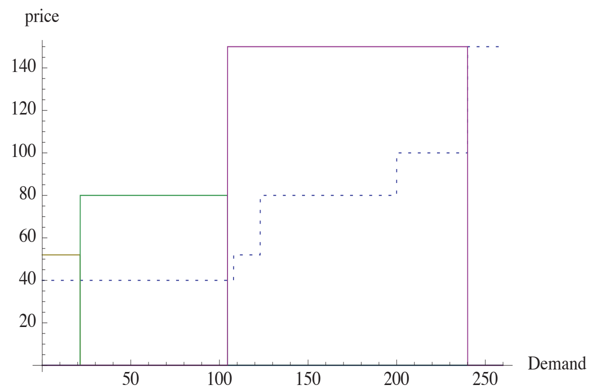
3. High demand. Take the case of $\theta = 170$. Here again DEH is the price setter in both cases of $\alpha = 0$ and $\alpha = 0.50$ bidding its lignite at 150. In the case of $\alpha = 0$, a competitor sell its gas production at 150 making a profit of $20 \times (150 - 80) = 1400$. In the case of $\alpha = 0.50$ a competitor sells both its NOME and remaining gas mixture at 150 making profit $25 \times (150 - 52) + 15 \times (150 - 80) = 3500$. The difference of profits in the two cases is 2100. Clearly, a competitor is better off using NOME in this case.

We observe that for low demand a competitor is indifferent with and without NOME, for medium demand he prefers the situation without NOME, and for high demand he prefers using NOME. In order to assess the total profit of a competitor under NOME we have to assume a distribution over the values of the demand and compute the expected difference of the profits in the two cases.[3] Note that even if we would offer the NOME lignite at DEHs cost (at 40), the competitors would still prefer no NOME regulation.

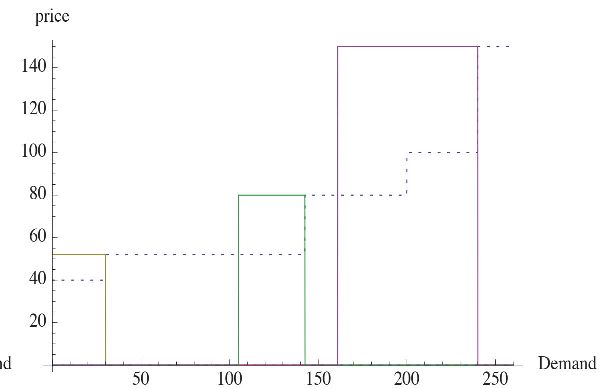
2.1.3 The price equilibria as a function of α

We already displayed the price equilibria for $\alpha = 0$ and for $\alpha = 0.50$. We add some figures for more values of α in an increasing order.

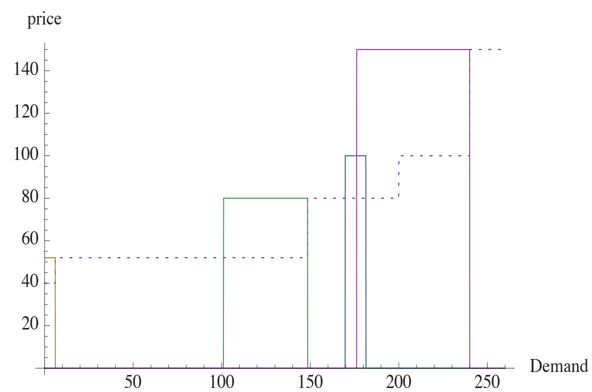
[3] In the case of the Greek market using data suggested by RAE, we can approximately assume that the time duration of high demand is 5h and the duration of medium demand is 7h. Hence the overall profit during medium and high demand using NOME is by far larger for the small providers.



(a) Market prices with NOME for $\alpha = 0.10$



(b) Market prices with NOME for $\alpha = 0.75$



(c) Market prices with NOME for $\alpha = 0.90$

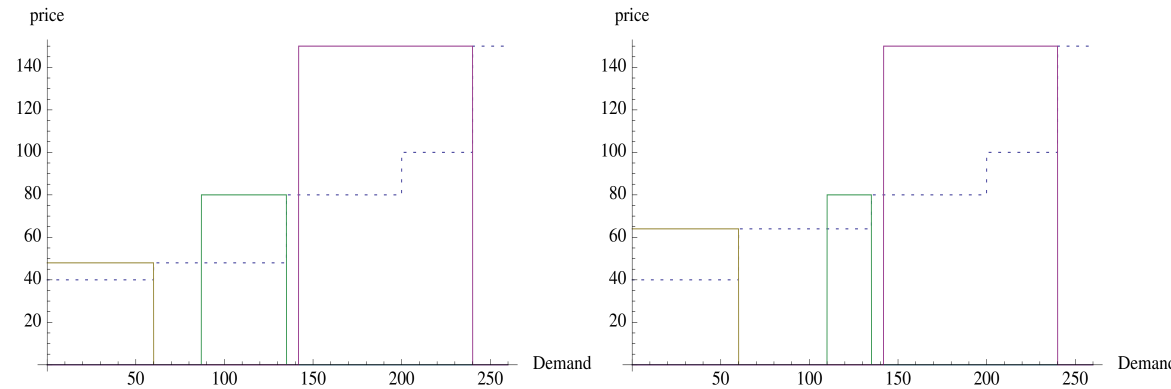
Figure 5: (a) We observe a new equilibrium at the NOME mixture marginal cost (at $p = 52$) that emerges for low demand. Otherwise the situation is similar compared to the case of $\alpha = 0$.
(b) We observe a longer range of values for low demand where the price is equal to the NOME marginal cost compared to the case of $\alpha = 0.50$ in Figure 1 (b). Also the range of demand values for which $p = P_{max} = 150$ is reduced.
(c) We observe an even larger range of demand with lower prices compared to the cases of lower α. A new equilibrium with $p = c_{I,h} = 100$ and price setters the gas producers emerges for medium/high values of demand but its effect is negligible.

Assuming that θ will be medium/low, then $\alpha = 0.50$ seems a reasonable value. For low demand it keeps the market price at the NOME mixture cost $c_N = 52$ instead of letting them rise to the price of gas $c_g = 80$ (under no NOME regulation). Similarly, for medium values of θ it keeps the prices at the cost of the gas instead of allowing them to climb up to P_{max} (the case of no NOME regulation).

These results suggest that for low values of $\theta < 90$ the cost c_N of the NOME mixture determines the market price. Hence lowering the price p_a of lignite will have a direct effect on the market price since $c_N = \phi c_g + (1 - \phi) p_a$. For medium values of the demand, the market price is determined by c_g and hence it seems not sensitive to p_a. To make this precise we need to consider the effect of p_a also on the range (the values of demand that support this equilibrium price) of the various equilibria, not just the value of the equilibrium price.

2.1.4 The effect of the price of lignite p_a

We consider the effects of p_a in the shape of the equilibria. We consider two extreme values $p_a = 40$ and $p_a = 60$. These imply $c_N = 48$ and $c_N = 64$ respectively.



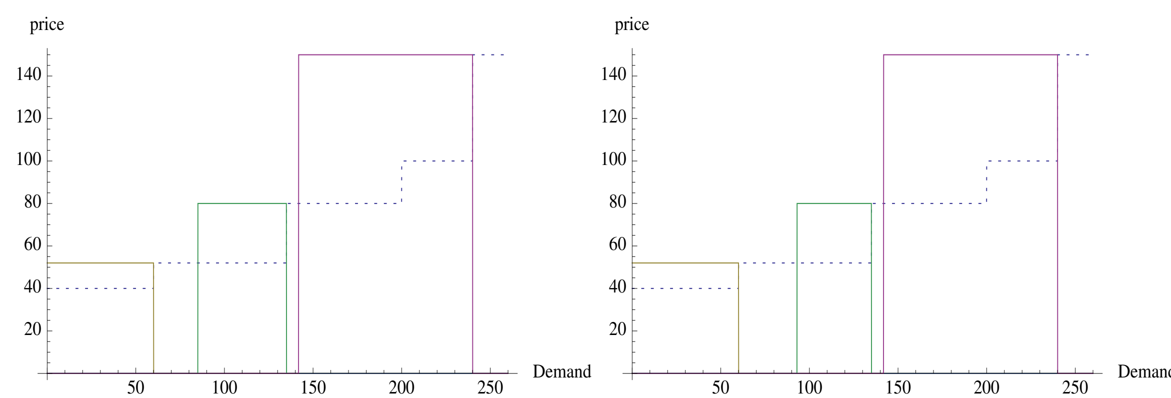
(a) Market prices with NOME for $\alpha = 0.50$ and NOME price for lignite $p_a = 40$

(b) Market prices with NOME for $\alpha = 0.50$ and NOME price for lignite $p_a = 60$

Figure 6: (a) The marginal cost of the NOME mixture is 48. (b) The marginal cost of the NOME mixture is 64.

Comparing these results we observe that increasing the cost of the NOME mixture has the adverse effect of raising the equilibrium prices for low demand, but it has the positive effect of reducing the range of medium demand for which we get an equilibrium at the price of gas. For instance, at $\theta = 100$ when $p_a = 40$ we get $p = 80$ whereas with $p_a = 60$ we get $p = 64$. This interesting observation suggests that if demand is medium, a higher p_a is beneficial. The intuition behind this observation is that having a larger cost for the NOME mixture, the gain by setting an artificially high price at c_g and selling less becomes less attractive

2.1.5 Asymmetry in the allocation of lignite to competitors of DEH



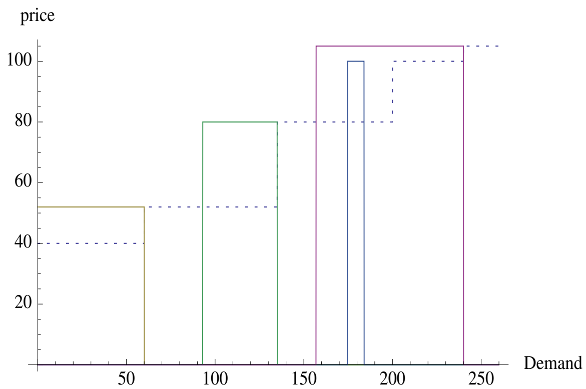
(a) The NOME lignite is allocated to the three competing providers of DEH in proportions 4:1:1

(b) The NOME lignite is allocated equally to two only among the three competing providers of DEH

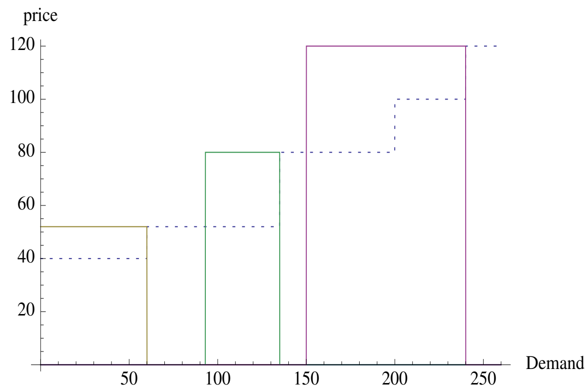
Figure 7: Market prices with NOME for $\alpha = 0.50$ and NOME price for lignite $p_a = 45$.

Comparing with the symmetric allocation in Figure 1 (b) we observe that there is almost no difference if competition due to the NOME product is reduced by one provider.

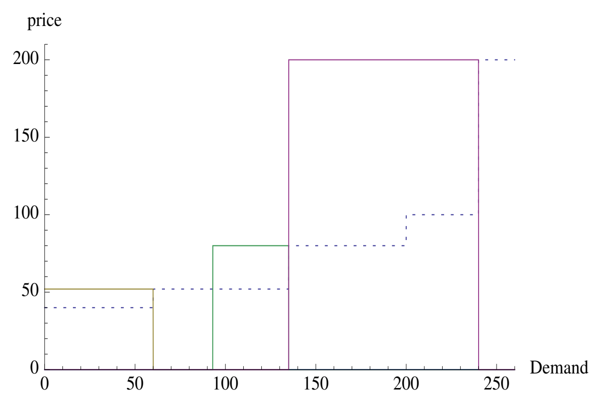
2.1.6 The effect of the price cap P_{max}



(a) Market prices with NOME when $P_{max} = 105$



(b) Market prices with NOME when $P_{max} = 120$



(c) Market prices with NOME when $P_{max} = 200$

Figure 8: Market prices with NOME for $\alpha = 0.50$, NOME price for lignite $p_a = 45$, and symmetric allocation.

We analyze the effect of P_{max} to the value of the price and also the range of the equilibria we have already encountered.

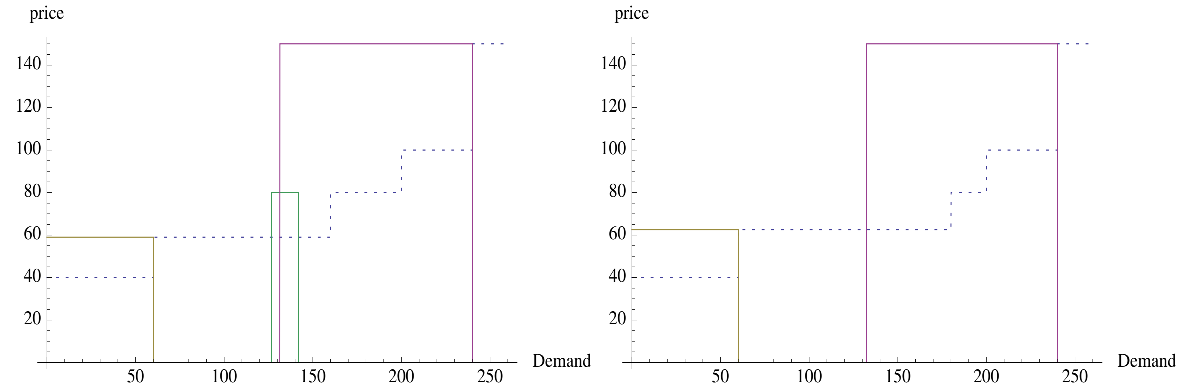
Comparing Figure 8 with Figure 1 (b) we observe that besides affecting the equilibrium price for high demand, decreasing P_{max} reduces the range for the price equilibria at $p = P_{max}$. But this effect is relatively minor as the analysis suggests.

2.1.7 The effects of changing the composition ϕ of the NOME mix

When we reduce ϕ and hence force the NOME mix to contain more gas, i) we reduce the available quantity of pure gas production by the competitors of DEH, and ii) the NOME mix is becomes more expensive.

There are two cases to consider. The first, say for $\phi = 0.60$, leaves some gas production free to compete after subtracting the gas capacity needed to combine in the NOME contracts.

The second leaves no extra gas to the providers. This occurs in our case when $\phi = 0.5$.



(a) Market prices with NOME for $\alpha = 0.50$, $\phi = 0.60$ (b) Market prices with NOME for $\alpha = 0.50$, $\phi = 0.50$

Figure 9: (a) In this case there are 6.666 units of gas capacity left per provider after allocating the gas needed to supply the maximum NOME mixture quantity. The equilibrium at the price of gas is generated by non-truthful price setting only by the DEH competitors.
(b) In this case there is no more gas capacity left per provider after allocating the gas needed to supply the maximum NOME mixture quantity. There is no equilibrium at the cost of gas since DEH being the only gas provider, when demand is above 130, it raises its bids to P_{max} which is higher than c_g.

Comparing Figure 1 (b) with Figures 9 (a), (b) we see that low values of ϕ have the negative effect of increasing the equilibrium price that is equal to the cost of the NOME mixture for low demand, but reduce significantly the range of medium demand where the equilibrium price is $c_g = 80$.

3 Forwards

In this section we will study the effects of forward contracts to the strategic behavior of the providers. Our model assumes that forward contracts have been set by the providers and are exogenous to the auction. More precisely we assume that provider i has a forward contract of an amount x_i to be provided to a customer at some agreed price $p_{f,i}$. We assume that x_i and $p_{f,i}$ are given and we don't study how these were determined initially (clearly $p_{f,i}$ is related to the uniform price auction results that determine wholesale electricity prices, but for simplicity we don't consider this effect).

Our model now works as follows. All providers are supposed to bid their energy production (including the amounts in the forwards) in the wholesale pool that operates under the uniform auction we have analyzed previously. This process will determine the equilibrium price for energy p^*. Now if q_i is the quantity that i is suppose to supply to the pool at the NE price p^*, then i will sell q_i at p^* and then buy back x_i at p^* for his customer. This is equivalent to selling a net of $q_i - x_i$ to the pool, but incurring a cost for the supply of the complete quantity q_i, $C(q_i)$. In this case his net profit is

$$\pi_i = p^*(q_i - x_i) - C(q_i) + x_i p_{f,i} \,. \tag{1}$$

Observe that the last term in the profit equation is fixed and does not depend on its strategy (he will make that revenue any way). Lets compare the profit in the case of zero forward. It is clear that in this case the term of the profit that multiplies the equilibrium price is larger and hence higher prices have weaker effects on profits under forward contracts. The situation becomes even more extreme if $q_i - x_i < 0$ in which case the provider makes a loss from his economic transaction with the pool, even if his costs are zero! Hence when $q_i - x_i < 0$ the strategy of the provider when bidding in the auction is to minimize the equilibrium price p^*. When $q_i - x_i > 0$ he benefits from high prices but his incentives to become the price setter are weaker than before (without forward contracts). Lets understand this more in detail.

Consider the case of DEH with $x = 0$, no use of NOME (for simplicity), and demand 40 units. The NE in this case is DEH to become price setter and bid all its production at the price of gas 80 while all other smaller providers bid truthfully. In this case it sells all $q = 40$ units at 80, since its bid will be accepted with priority since it has smaller MC than the other bids at 80. The alternative for DEH is to be truthful and sell more at a potentially lower price, making more profit than before. In this later case, the market price will be the cost of lignite 40, and hence DEH makes 0 profit. To prove that DEH is acting strategically at the NE we need to check that its profit by doing so $p^* \times q - C(q) = 80 \times 40 - C(40)$ is higher than $40 \times 40 - C(40)$, which is true. Note that the difference in profits is $80 \times 40 - 40 \times 40$ (we have omitted the term $x_i p_{f,i}$ which plays no role when comparing differences).

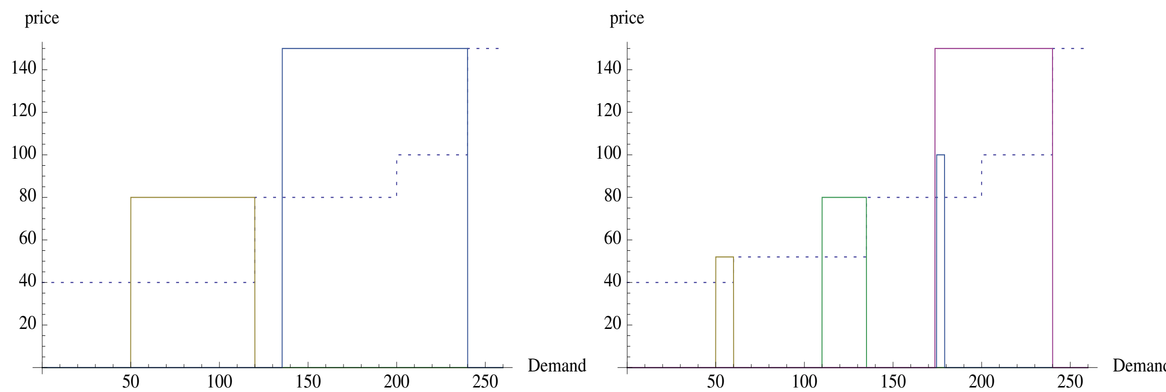
Consider now the case of DEH having a forward contract x. Then the difference of the profit in the cases of acting strategically versus being truthful is $80 \times (40 - x) - C(40) - (40 \times (40 - x) - C(40)) = (80 - 40)(40 - x)$. Note first that this difference decreases in x. And for $x > 40$ it reverses sign, making truth telling the optimal strategy! This observation generalizes in the case of more general NEs where some provider finds it profitable to be the price setter in the case of no forward contracts. If we add such contracts, this deviation by setting a high price becomes weaker in profits and it may become suboptimal to the case of being truth telling. Hence incentives for price setting become weaker under forward contracts, and equilibrium prices tend to become lower.

Under arbitrary forward contracts the analysis of the NEs becomes more complex. If we allow providers to have larger contracts than their production ($q_i < x_i$, become net buyers) then we get complex equilibria where providers may find it best to bid lower than their true MC in order to keep equilibrium prices low. This makes the complete analysis of the general case very hard and our simple NE characterizations offered in the previous sections do not apply any more. The good news is that the analysis remains the same if we don't allow strategic under-bidding (bidding below MC) and if we are in the range of $q_i - x_i > 0$. In this case we need to check similar conditions for price setting as before, with the difference being that now the quantity sold at the high price in the pool is $q_i - x_i$ instead of q_i. This in most cases reduces the range of the various equilibria compared to the case of having no forward contracts, but it also produces some new ones. This later case occurs when a set of bids was not an equilibrium (with no forward contracts) because a non-price setter was finding it more profitable to deviate from his truthful bids, and then, having a forward obligation, such a deviation becomes not any more optimal.

In the next section we analyze the market assuming forward contracts and compare to the cases where no such contracts exist.

3.1 Analysis with forward contracts

We consider first the case of forward contracts provided only by DEH, with $x_1 = 50$. In Figure 10 (a) we show the equilibria without NOME and in Figure 10 (b) we show how these are rearranged when we use NOME with $a = 0.50$. These figures are in analogy to Figure 1 (a) and (b).



(a) Market prices without NOME when only DEH has a forward contract $x = 50$

(b) Market prices with NOME and $a = 0.50$ when only DEH has a forward contract $x = 50$

Figure 10: The existence of the forward contract reduces the range of high equilibrium prices due to strategic price setting.

Lets compare first Figure 10 (a) with Figure 1 (a). We observe that prices drop uniformly, the range of equilibria where DEH is the price setter is reduced, and for a large range of medium demand, prices dropped to 80 from 150. Also for demand below the value of the forward contract, the optimal policy for DEH is to bid its MC. Similar observations hold when comparing Figure 10 (b) with Figure 1 (b). Here we observe the emergence of a new high price equilibrium at 100, for demand equal to 175. This is due to the competitors of DEH setting the high price, and was not possible without forward contracts because then DEH was find it more profitable to move away from being truth telling (and now this is not anymore the case, see remark in previous section).

Lets compare the case of having a forward contract, with and without NOME. As we see, under NOME with forward contracts, the range of high price equilibria is further reduced. For a larger range of medium demand values we get a price equilibrium at 80 and at 52. Below the amount of the contract prices drop to MC.

3.1.1 Incentives of small providers

An interesting analysis concerns the incentives of the competitors of DEH to deploy NOME when there are forward contracts (in our case, for simplicity, only from DEH's side). Consider the case of medium demand $\theta = 120$ without NOME. Our analysis shows that in this range of demand values the competitors of DEH make zero profit. If demand is 110, then DEH is the price setter at 80 and serves all the demand from its lignite production. If demand is 130, then the equilibrium is again at price 80, and competitors use their gas production to supply part of the 10 units left after using all the lignite production, at zero profit.

Consider now the case of using NOME as in Figure 10 (b). Then at $\theta = 120$ the equilibrium is due to price setting by some competitor of DEH, while DEH and the others remain truth telling. In this case DEH sells all its lignite production of 60, and the remaining 60 units are filled by production using NOME mixture, sold at the price of gas. Now all the competitors make strictly positive profits compared to the case of not using the NOME capability. Hence we reverse the case investigated in Section 2.1.2, where for medium demand the smaller providers using NOME were worse off.

When demand gets a bit higher, the smaller providers are still earning positive profits under NOME but they may be worse off compared to the case of not using NOME. For instance, when $\theta = 150$, without NOME DEH is the price setter at 150 with its lignite bid, where each competitor sells his complete gas production at the above price, making a profit of $20 \times (150 - 80) = 1400$. Under NOME, the NE is a truth telling one, and here each competitor makes his profit by selling its NOME mixture at the price of gas, making $25 \times (80 - 52) = 700$. Hence due to the lower equilibrium price their profits are reduced. Of course, for larger demand values in both cases the equilibrium prices are at P_{max} and competitors make substantially more profits under NOME since they sell both their NOME and gas products at 150.

We plot the profits of the small providers from the auction as a function of the demand in Figure 11.

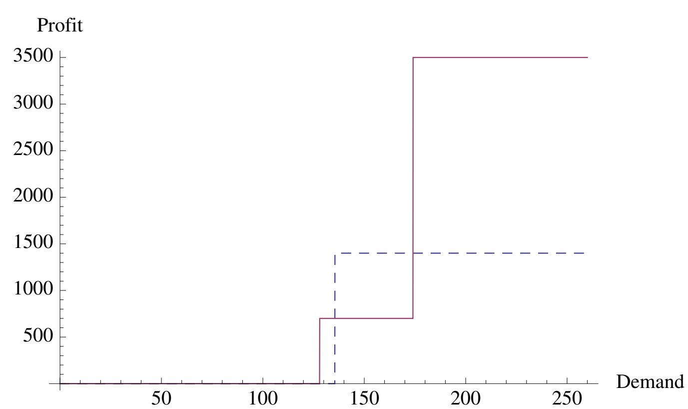


Figure 11: Profits from the auction for a small provider. The dashed line corresponds to the case without NOME and the normal line to the case with NOME. In both cases DEH has a forward of $x = 50$ and in the case of NOME small providers have a forward of 25 each. In the above profit functions we don't include the profits/losses from the forwards. The existence of forward contracts has influenced the equilibria and hence the market prices.

This analysis suggests that the only demand interval where small providers regret the adoption of NOME is $[135.5, 174]$. But even in this interval they are profitable. And if demand is above 174, they make significantly larger profits.

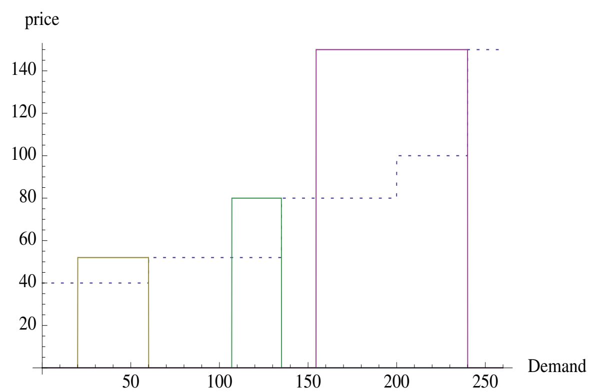
Comparing Figures 4 and 11 one observes that the existence of forward contracts, especially from the part of DEH, narrows the range where the use of NOME is not preferable. In any case, one has to consider the expected distribution of the demand to make any firm conclusions.

3.2 Sensitivity under different forward contract quantities

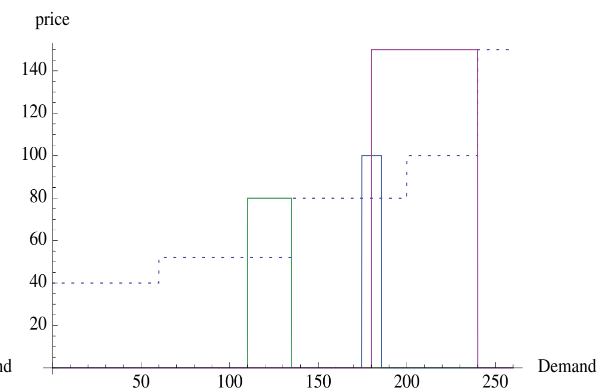
To complete our analysis on the effects of forward contracts to the outcomes of the auction we consider the effects of increasing quantities of forward contracts for both DEH and its competitors.

Observing Figures 12 (a), (b) we note the positive effects of the forward contracts of the large provider in lowering equilibrium prices.

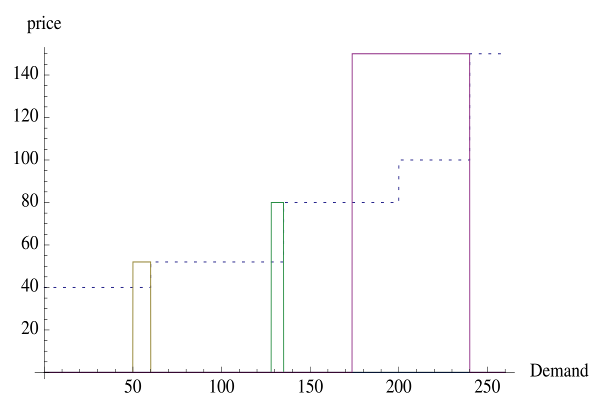
We next show the effects of forward contracts from the side of the smaller providers that use NOME. We show in Figure 12 (c) the case of such providers having forward contracts equal to the size of their NOME product, i.e., equal to 25 (assuming the rest of the parameters as before).



(a) Market prices with NOME and $\alpha = 0.50$ when DEH has a forward contract $x = 20$



(b) Market prices with NOME and $\alpha = 0.50$ when DEH has a forward contract $x = 60$



(c) Market prices with NOME and $\alpha = 0.50$ when DEH has a forward contract $x = 50$ and each of its competitors a forward contract of 25.

Figure 12: (a) Observe that the high price equilibria are not reduced as much as when $x = 50$. (b) We observe that when the forward contracts are larger, DEH has a much lower incentive to become the price setter, and the equilibrium prices are near marginal cost. (c) Except for demand in the high range, equilibrium prices are almost at marginal costs.

Comparing Figure 12 (c) to Figure 10 (b) we see that assuming forward contracts for the smaller providers has a rather negligible effect on the equilibrium prices, except for a narrow demand range between 110 and 130. This shows the importance of forward contracts when these are from the part of the dominant provider.

4 Adding practical constraints

4.1 Assuming that lignite production has large "turn-off-and-on" costs

The previous analysis of the Greek energy market is based on the assumption that providers do not have 'turn-off-and-on' costs (TOC costs). These costs occur when a facility needs to be shut down and turned on again and is known to occur for specific technologies. In our case, the power generation based on lignite production has exhibits these TOC costs. Lets understand where these costs affect the previous analysis.

Our uniform price auction model helps is predicting the strategic behavior of a power producing firm in an unregulated environment *for a given demand* when it can flexibly price its power generation by taking into account its actual costs in an oligopoly context. Hence our previous analysis regarding the calculation of the equilibria holds in case there a single demand and the firm optimizes its strategy for this specific demand. These are the plots we have shown in the previous figures, where we investigate the strategic behavior of the providers in the oligopoly market for all possible values of (inelastic) demand. The situation is different if demand stays not constant during long periods of time, but varies, e.g., it changes possibly at every hour (the time slot) in a day.

Consider the case of time slots k, $k+1$ and $k+2$, where demand is high, low and high respectively. Assume also that market prices are expected to be p_H, p_L, p_H respectively, and that the average cost of a lignite facility by DEH with large TOC cost is $p_L < c < p_H$ (the average cost when the facility is on). Assume also that this facility is selected to contribute power in slot k as the result of the auction at time k. Then DEH, expecting to have its facility again selected during $k+2$ when $c < p_H$, might bid this facility's production during $k+1$ below cost in order to keep it on for $k+2$ and not incur the TOC cost. Hence making a loss from the market during $k+1$ but keeping the facility operational might be a better choice for DEH (or for any other provider operating facilities with substantial TOC costs).

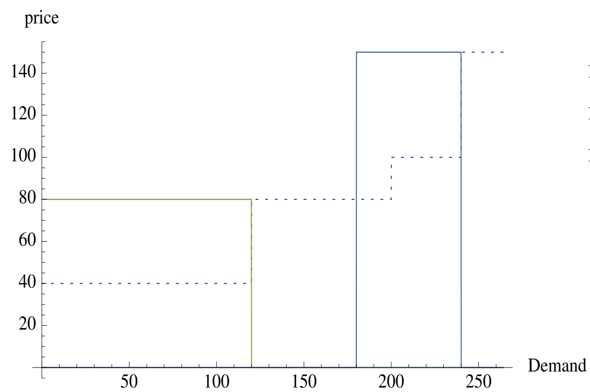
This example suggests that a provider might not bid at each time slot in an independent manner. Our analysis provides the right results in the case of a repeated auction at different time slots if the strategic choices of competitors are made in a way that does not considers the previous and the next states of the system, hence if strategic choices are made independently of the past and the future. A crucial question that remains is how to address such TOC issues and to what extend these influence our analysis. We can state with certainty that creating a model of strategic behavior for providers that takes into account the past and future information about demand and the TOC costs is intractable. Hence we need to propose some simplifications which will make it tractable.

Our proposal is the following. Assuming that lignite production is mainly the one associated with large TOCs, we consider improbable for DEH to strategically use it for price setting at some market equilibrium **if this involves substantial reduction in lignite capacity sold**. Hence we assume that due to high TOC for lignite, DEH must keep most all its lignite generation active at all times, and does not act strategically to reduce it (unless actual demand is below this value in which case DEH cannot do anything to prevent it[4].

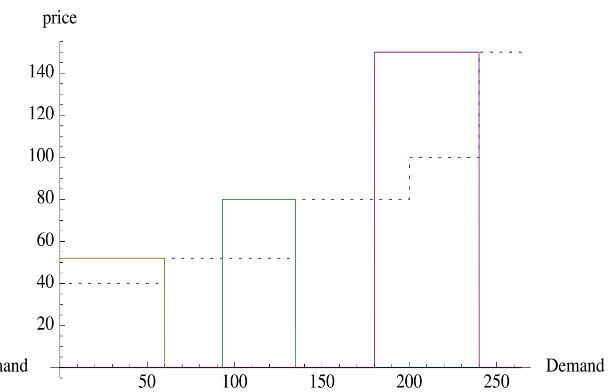
[4] A possibility is to sell it below cost in order to generate the extra demand required.

4.2 Adding TOC constraints in the market

To implement our heuristic mentioned earlier *we disallow the occurrence of equilibria where DEH sell its lignite at prices above the cost of gas.* This is because such a price setting intrinsically implies that a significantly small quantity of the lignite product is sold at a high price. On the other hand, price setting of lignite at the price of gas or below (at the price of the NOME mixture) will not reduce the potential for the maximum capacity of lignite to be sold since because of its lowest MC it will have priority in filling the existing demand.



(a) Market prices without NOME ($\alpha = 0$) and TOC



(b) Market prices with NOME for $\alpha = 0.50$ and $\phi = 0.80$ and TOC

Figure 13: (a) Market prices without NOME ($\alpha = 0$) when DEH does not perform price setting with its lignite above the price of gas. This is the analogous of Figure 1 (a) where the effects of high TOCs were not considered. Comparing the two one observes a substantial reduction of the region where the equilibrium price was P_{max}, with a simultaneous increase of the region with equilibrium price the price of gas.
(b) Market prices with NOME for $\alpha = 0.50$ and $\phi = 0.80$ when DEH does not perform price setting with its lignite above the price of gas. This is the analogous of Figure 1 (b) where the effects of high TOCs were not considered. Comparing the two one observes a substantial reduction of the region where the equilibrium price was P_{max}, the new price being power and equal to the price of gas. The new equilibrium for medium demand (say for $\theta = 110$) is due to DEH setting its lignite price at the price of gas. When demand gets higher, say at $\theta = 120$, there are similar equilibria where the other providers sell their NOME product at the price of gas.

This reduces the possible equilibria we encountered before. For instance, in the case of no NOME in Figure 1 (a), for $\theta = 110$ DEH acts strategically and performs price setting at P_{max} selling only 50 units of its lignite production (its competitors sell the total 60 units of their gas also at P_{max} by bidding truthfully). Applying our heuristic this equilibrium is not possible.

The resulting pruning of equilibria is shown in Figures 13 (a), (b). We observe in both cases that prices for medium range demand are lower as expected since DEH is bidding in a way to ensure that most of its lignite production is always consumed. There are two striking properties that one notes now.

First, both in the case with and without NOME, the high price equilibria are not changed. This is because in both cases at the initial demand range corresponding to this equilibrium, price setting is caused by DEH setting its gas at P_{max}. Hence the existence of NOME does not influence that part of the demand.

Second, under NOME the demand range where the price is the price of gas is reduced. Due to competition between providers using NOME, this price drops from the price of gas 80 to the marginal cost of NOME for demand below 93. Here the marginal cost of the NOME mixture determine the market price.

4.3 Incentives of providers to adopt NOME

The situation is very simple now. For a demand in the range $93 < \theta < 135$ they make strictly more profit than before since they sell all their NOME mixture at the price of gas. The same holds for demand values above 135 up to 180, where the equilibria are truth telling and the price is the price of gas since the price is the same as in the case without NOME, but now they make positive profit by selling the less costly NOME product. Then, when the price rises to P_{max} they make again more profit than before since they sell both the NOME product and the remaining gas at price 150. **Hence the NOME providers are for all values of demand ≥ 93 strictly better off than in the case of not using NOME**.

Our numerical values using current data from RAE for the expected scaled demand values in the market are the following[5]: i) low demand: 101 units, occurs 7/24 of the time, ii) medium demand: 132 units, occurs 12/24 of the time, iii) high demand: 164, occurs 5/24 of the time. We observe all demand values are in the range where providers are better off using NOME than without. And the profit is made by selling their NOME product at the price of gas.

We next compute explicitly the profits of providers with and without NOME in Figure 14.

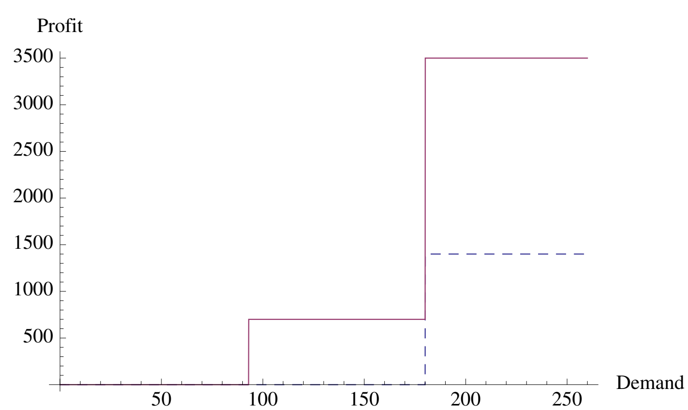
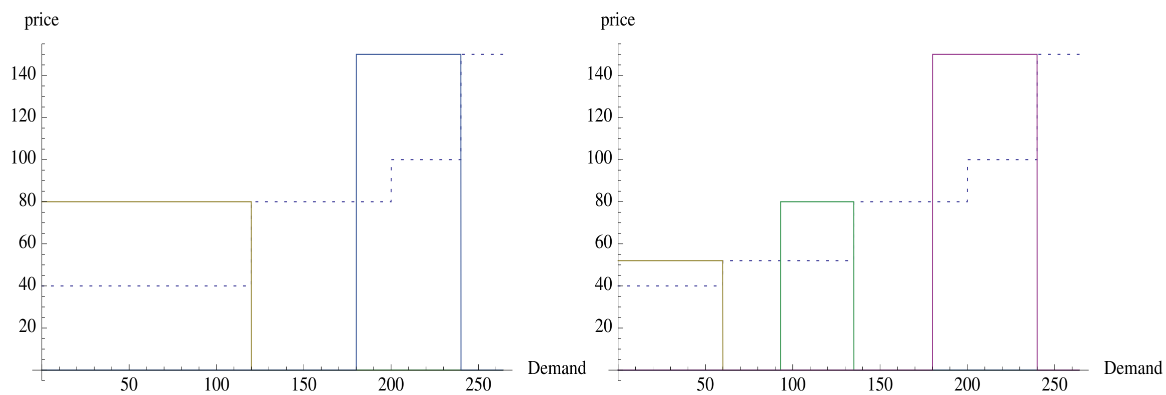


Figure 14: Profits from the auction in the deregulated market for a small provider assuming that DEH is not price setting its lignite production at prices above the price of gas due to the existence of TOCs. The dashed line corresponds to the case without NOME and the normal line to the case with NOME. We observe that the profits of the providers under NOME are uniformly higher, and strictly higher when demand is above 93.

[5]The demand that will go to the auction is calculated by subtracting from the actual demand the average value of the power generated by alternative source, estimated to be of the order of 1000MW

4.4 The effect of forward contracts

We consider the case of forward contracts provided by DEH with $x = 20$ and for the rest of the providers with $x = 10$. In Figure **??** we show the equilibria without NOME and in Figure **??** we show how these are rearranged when we use NOME with $a = 0.50$.



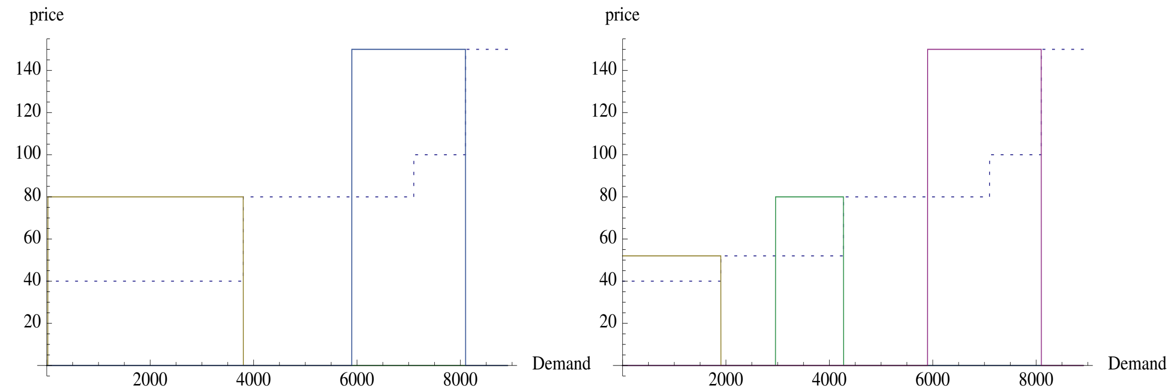
(a) Market prices without NOME (b) Market prices with NOME

Figure 15: (a) Market prices without NOME when only DEH has a forward contract $x = 20$ and the rest of the providers forward contracts $x = 10$. The existence of the forward contract reduces only the range of equilibrium prices at very low demand where DEH was price setting its lignite production at the price of gas.
(b) Market prices with NOME when only DEH has a forward contract $x = 20$ and the rest of the providers forward contracts $x = 10$. The existence of the forward contract reduces only the range of equilibrium prices at medium demand where DEH was price setting its lignite production at the price of gas.

These figures are in analogy to Figures 13 (a), (b). We observe that the range of demand where the NOME providers are strictly better by using NOME is reduced (must be above 107 instead of 93).

4.5 Using current market data

So far our model for the market used scaled values for the various quantities, corresponding to the state of the market the year before. These scaled values are expected to change as time passes, but not by much. We show the unscaled graphs that correspond to Figures 13 (a), (b) using updated information about the future power generation. We observe that the results are very similar, and hence small changes of the parameters are not affecting our key observations so far.



(a) Market prices without NOME and TOC (b) Market prices with NOME and TOC

Figure 16: We use actual market values instead of scaled.

We observe that for all values of demand (low demand[6] = 3200MW, medium demand = 4200MW, high demand = 5200MW) the NOME providers are better off in terms of profits comparing to the case without NOME.

4.6 Changing the values of ϕ and α

We show the market prices for different values of ϕ and α. We choose $\phi = 0.65$ and $\alpha = 0.30$. The results are in the following figures.

In Figure 17 we see the effects of reducing both α and ϕ. We observe that the NOME providers keep making for all target demand values of 3200, 4200, 5200 strictly higher profit than in the case without NOME, but this profit is reduced since they incur a higher cost and the maximum amount they can sell is less. At these demand values the market price continues to be at the cost of gas.

The profits of the NOME providers with and without NOME are shown in Figures 18 (a), (b) for $\phi = 0.80, \alpha = 0.50$ and for $\phi = 0.65, \alpha = 0.30$ respectively.

[6]After subtracting 1000MW corresponding to inelastic generation (dollar, wind, water-related).

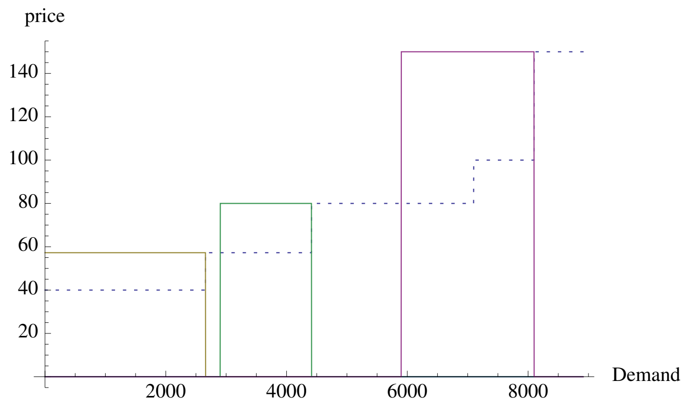
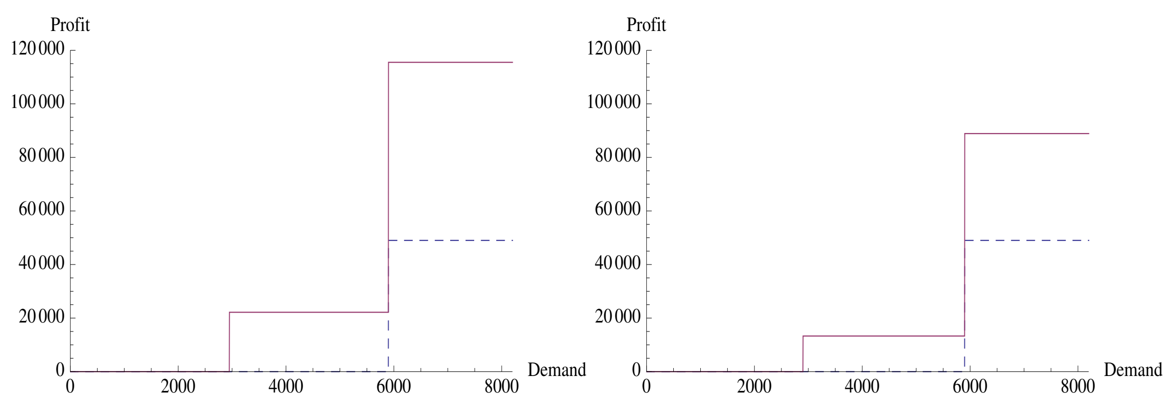


Figure 17: Market prices with NOME for $\alpha = 0.30$ and $\phi = 0.65$ corresponding to future expected market power generation parameters under the heuristics that capture TOC costs. We observe the equilibrium at the higher price of 57.25 (the cost of the NOME mixture) at lower demands. The rest of the prices remain the same. DEH has a larger range of making profit by selling its lignite at the price 57.25 of the NOME mixture. The other providers continue to make profit during low to high demand by selling their NOME product at the price of gas, which is reduced since they need to mix lignite with a higher portion of gas and the total amount of NOME product is reduced (since $\alpha = 0.30$ instead of 0.50).



(a) Profits from the auction in the deregulated market for a small provider when $\phi = 0.80, \alpha = 0.50$ and TOC

(b) Profits from the auction in the deregulated market for a small provider when $\phi = 0.65, \alpha = 0.30$ and TOC

Figure 18: (a) Profits from the auction in the deregulated market for a small provider when $\phi = 0.80, \alpha = 0.50$ DEH is not price setting its lignite production at prices above the price of gas due to the existence of TOCs. The dashed line corresponds to the case without NOME and the normal line to the case with NOME. We observe that the profits of the providers under NOME are uniformly higher, and strictly higher when demand is above 2950.
(b) Profits from the auction in the deregulated market for a small provider when $\phi = 0.65, \alpha = 0.30$ DEH is not price setting its lignite production at prices above the price of gas due to the existence of TOCs. The dashed line corresponds to the case without NOME and the normal line to the case with NOME. We observe that the profits of the providers under NOME are uniformly higher, and strictly higher when demand is above 2900.

4.7 The effects of asymmetries in the NOME allocation

We show the case of a single provider getting the NOME capability in Figure 19.

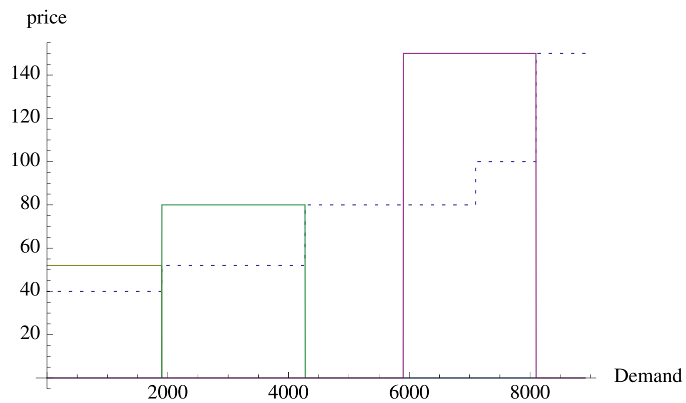


Figure 19: Market prices with NOME for $\alpha = 0.50$ and $\phi = 0.80$ when a single provider gets 100% of the NOME capability. We observe that the equilibrium at the price of gas extends for lower demand values.

4.8 The effects of p_a

In Figure 20 we show the effects of higher prices on the price p_a of lignite at the NOME auction.

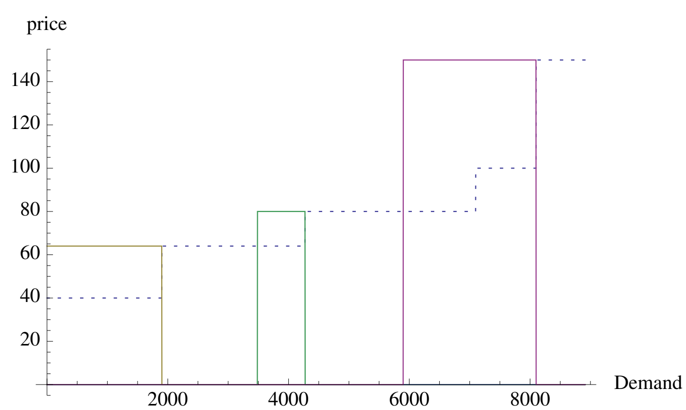


Figure 20: Market prices with NOME for $\alpha = 0.50$ and $\phi = 0.80$, when $p_a = 60$. For $\theta < 3484$ the price drops to the price of the NOME mixture which is higher due to the higher p_a. Hence the the equilibrium at the price of gas 80 starts at a higher demand than when $p_a = 45$, but for lower demand the price is higher.

5 Linking the wholesale (pool) market to the NOME auction

In this section we discuss the solution of the two-stage game that corresponds to the combination of the initial NOME auction with the subsequent uniform auction of the pool. In the first a provider buys the NOME capacity he will use subsequently in the uniform price auction to compete with the other providers. The issue we need to investigate is how these auctions are related and how this affects the strategies of the players in both.

Chapter 5 of our Report focuses also in these issues, and produces the guidelines for selecting the mechanism for allocating the NOME quantities to the Es. It uses the findings of this section that Es like to buy the maximal amount of lignite in stage I, since they make clear profits in stage II.

So far we did not address this relation in defining the provider strategies in the uniform price auction. In particular, we assumed that a provider has access to a virtual power plant with the average operating cost of the NOME mixture, say 52, the size of which was already pre decided (is exogenous to the auction). By bidding its production at a certain price, we implicitly assumed that if the bid is accepted, the power plant will operate and provide the requested power at a cost of 52, and if the bid is not accepted, then the plant will not operate and there is no cost involved. But is this a valid assumption?

In reality a provider at the NOME auction will purchase a quantity of NOME capability, which is statically allocated to him during all times for the next period of the contract[7] And each provider pays DEH for the lignite irrespectively of how he will use it later on. This implies that when competing in the market, the cost of the NOME mixture is a sunk cost (actually the cost of the lignite). Hence it should not affect his decisions on acting strategically and placing bids. The cost we should use to calculate the best response of a player in the auction should be zero!

Lets define the two stage game precisely.

- Stage I: The NOME allocation.
  - Known: the price p_a of lignite and the maximum quantity $\bar{q}$ a provider is allowed to buy.
  - Player i decides on the lignite quantity $q_i \leq \bar{q}$ (its strategy) to purchase at the given price p_a.
- Stage II: The uniform auction were providers sell their products purchased in Stage I.
  - Known: For each player i, the quantity of NOME mixture available q_i/ϕ, and full information about the capabilities of the other players.
  - Player i decides on his bids to maximize profits.

The previous definition of the full game implies that i) the strategy at the second stage of the game does not depend on the cost incurred in the first stage, but only on the quantity of NOME that has been made available (is a subgame-perfect equilibrium), and ii) the strategy of the first stage depends on the profits acquired in the second.

[7]We assume that NOME contracts are not options that can be exercised at a certain time or not. They represent contractual obligations for DEH to provide daily its part of the lignite contract power for the price determined in the NOME auction.

A helpful paradigm to understand this is that at the first stage of the game a virtual power plant is been build by player i, that **has to operate continuously at power level** q_i (cannot be shut down), and consumes for each unit of NOME power ϕ units of lignite with cost p_a and $(1-\phi)$ units of gas with cost c_g. Then the second stage of the game is played assuming the existence of this investment from the part of player i.

These remarks imply the following conditions on the Nash equilibria of the game

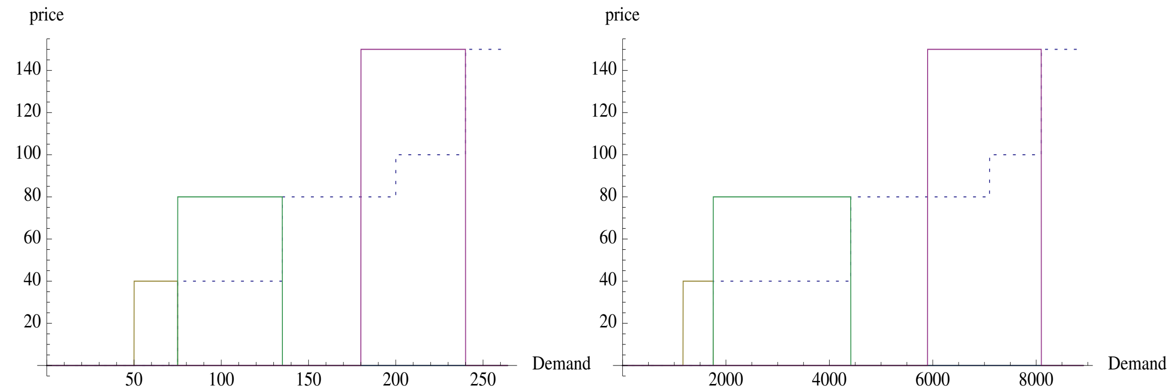
- The actual cost parameter we should use in our uniform auction model for the NOME mixture is 0. This is because the cost of the virtual power plant is sunk.
- Player i should never buy in stage I a quantity q_i higher that the quantity sold in the market at the equilibrium of stage II.

By assuming in our initial modeling of the uniform price auction that the cost of the NOME mixture is 52 , we implied that when such a product is sold it costs 52 to the provider (which is correct, because this is the cost of the virtual power plant), but when it is not sold (say because of low demand, or because the bid price was too high) it costs zero (which is not true since the provider needs to pay DEH for the NOME product anyway and also supply his corresponding part of the gas production).

5.1 The Nash equilibrium

We will prove that for the numerical values (for the demand and the costs) considered in our previous analysis the sub game perfect equilibria are for the providers to purchase the maximum possible quantity allowed in stage I. We will also show the actual price equilibria that result during stage II.

Suppose $\alpha = 0.50, \phi = 0.8$, and that each provider purchased the maximum possible $\alpha/3$ during stage I. We show the equilibria of the uniform auction as a function of the demand considering the cost of the NOME mixture being sunk (i.e., zero) in Figure 21. As we show in our analysis, this diagram accurately depicts the market prices ONLY for $\theta \geq 75$. For lower demand values it only show sub game perfect equilibria that occur in stag II. But these



(a) Market prices in the stage II game (b) The similar setup for actual market values

Figure 21: (a) Market prices in the stage II game for $\alpha = 0.50$ and $\phi = 0.80$, when the cost of NOME is sunk (zero) and each provider is allocated $\alpha/3$. Same as before we assume that DEH is not price setting its lignite at prices above the price of gas. For very low demand the equilibrium drops at the cost of NOME (zero). When $\theta = 60$ the NOME providers are performing price setting NOME product at the cost of lignite. Then for $75 \leq \theta \leq 135$ DEH is price setting its lignite at the price of gas. When $123 \leq \theta \leq 135$ some NOME provider may also perform price setting by bidding his NOME product at the price of gas. The rest of the equilibria are as before. As we show in our analysis, this diagram accurately depicts the market price equilibria (stage I and stage II) ONLY for $\theta \geq 75$.
(b) The similar setup for actual market values when $\alpha = 0.30$ and $\phi = 0.65$. As before, this diagram accurately depicts the market price equilibria (stage I and stage II) ONLY for $\theta \geq 1774$.

We will prove now that these prices are also price equilibria for the complete two-stage game for demand higher than 75.

To prove that we note that (see comments in Figure 21)

1. For $75 \leq \theta \leq 135$ there is an equilibrium where DEH is the price setter and the rest of the providers sell ALL their NOME capacity (all the quantity q_i available from the first stage).

2. At any such demand value, the total profit (in the complete game) of provider i is $q_i(80 - p_a)$. Assuming that $80 > p_a$, the optimum strategy of provider i in stage I is to maximize q_i. Hence he should buy the maximum possible, i.e., $\alpha/3$, which is the case we have assumed when computing the equilibrium prices at stage II.

3. For larger demand values, NOME providers sell always all their NOME capacity, and make a positive profit from each unit sold in the market. Hence again they will maximize the capacity bought in stage I.

We observe that **Figure 21 accurately depicts the market prices ONLY for** $\theta \geq 75$. For lower demand values it only show sub game perfect equilibria that occur in stag II. But since in these equilibria some NOME provider (the price setter) sells less than his available quantity $\alpha/3$ purchased in stage I, it cannot be a Nash equilibrium of the complete two stage game. Hence we don't know the actual price equilibria for $\theta < 75$, which may consist only of mixed strategies. We observe in Figure 21 (b) that **we can predict accurately the market**

price equilibria only for $\theta \geq 1744$. We believe that this is sufficient since currently the low demand value in the market is around 3200MW.

5.2 Strategy issues

In the previous section we analyzed the two stage game assuming that participants act rationally. Specifically, we assumed that DEH in stage II will be the price setter at the price of gas when $75 \leq \theta \leq 135$ in Figure 21 (a), since it is to its advantage to do so. The fact that price will rise to the price of gas is beneficial to the other providers since they can sell their NOME product at the price of gas and make pure profit. But what about if DEH acts in a predatory way and bids its lignite at the cost of NOME or below? Then clearly the other providers will not make any more profits when the demand is in that range. This will discourage them to participate in the NOME auction and purchase some $q_i > 0$. Hence the risk of DEH pricing its lignite at the cost of NOME or below reduces the incentives to participate at the NOME auction. In theory this is not a credible threat since if the other providers overlook the threat and purchase NOME capacity, in stage II DEH will act as expected in order to maximize its profits and set the price at the cost of gas. Nevertheless, this risk is worth mentioning.

In assessing this problem in more detail we observe in Figure 21 (b) that this margin squeeze strategy by DEH can have an effect only for low and medium demand. During high demand ($\theta = 5200$) the NOME providers make definitely profit. This occurs only 5/24 of the time and hence the net profit calculation shows a net loss for the NOME providers.

A possible way to remedy this situation is to reduce the benefits of DEH if it follows this strategy and add some security to the other providers. There are several possibilities:

1. Allow in the NOME auction to purchase the NOME capability for specific time intervals (for high demand). This clearly changes the picture and our analysis applies for these higher demand values. There is clear net profit obtained by the NOME providers without any risk.

2. Regulate DEH and prohibit strategic bidding of the lignite that is lower than what would be more profitable for DEH. The effects of such a regulatory measure are obvious.

3. When demand is low and medium, allow the providers to "separate" their NOME product into lignite and gas, and bid only the lignite part with no obligation to mix it with gas. This makes them more competitive and ensures a very low loss at worse case which will be compensated by the large profit made at the high demand. Simple profit calculations show that in the case of Figure 21 (b) NOME providers will make a net profit of 1200 if DEH prices its lignite at cost during medium and low demand. Adding this "security" to the NOME contracts ensures that providers will participate in the NOME auction and it will reduce the incentives of DEH for price manipulation. These bids could be made conditional on DEH bidding its lignite at cost.

6 A preliminary analysis of the retail market in the short-run (in the aftermath of implementing NOME)

We like to analyze some interesting properties that arise in a market that consists of a wholesale and a retail market where NOME providers are selling all their NOME products using

OTC contracts in the retail. We will show that there is a certain 'decoupling' of the wholesale mandatory pool market and of the retail OTC market.

A key assumption of the analysis is that the an E provider is able to sell ONLY his NOME production in the retail market via OTC contracts, and he is not acting as a general reseller of energy bought from the pool. This constrained behavior is assumed to take place in the aftermath of the NOME regulation, where each E provider will make sure to fulfill the restriction of the regulation, and will start from an empty customer base. Eventually each E will be able to compete freely in the retail market with I and the other Es selling more than his own NOME production, buying from the wholesale market pool. This more general competition that will take at a later stage is investigate in Chapter 4 of the Report.

For simplicity we assume that the competing providers are DEH (the incumbent I) and a unique provider E which has acquired a NOME product in the NOME auction. Our assumption for E is that his initial power production was using gas. We use q_N to denote the quantity of the NOME product available by provider E and assume that lignite cost in NOME is p_a. Let $l(q_N), g(q_N)$, $l(q_N) + g(q_N) = q_N$, denote the lignite and the gas parts of the q_N NOME product quantity (i.e., $l(q_N) = \phi q_N, g(q_N) = (1 - \phi)q_N$).

Our specific assumptions for the structure of the market are:

- **Whole sale market mechanism**. This mechanism is used to define the wholesale market price p_W and the scheduling of the physical plants of the providers. More specifically:

  1. All providers (I, E) offer their supply functions (bids) for all their plants. In particular, I includes lignite plants that offer the power allocated to the NOME contract, and E offers his complete gas production, including the part that corresponds to his NOME obligation.
  2. The mechanism operator uses the rules of the mechanism (possibly a uniform price auction) coupled with technical scheduling requirements to determine the plants that will be asked to provide power for the given demand θ. He also determines to clearing price which p_W which is the wholesale price of the market. His constraints are that he must activate at least $l(q_N)$ quantity of lignite production for I and at least $g(q_N)$ of gas production for E (a way to implement the NOME regulation).
  3. The providers follow the resulting schedule and provide power sold at p_W.

- **Retail market mechanism**. Here I, E purchase power from the wholesale market and sell it to the retail market. The key assumptions are:

  1. E: He sells all his q_N in the retail market at price p_R^E. The lignite part $l(q_N)$ of his contract q_N is bought from the wholesale market at price p_a (hence I is charged the difference $p_W - p_a$ for that quantity).
  2. I: Needs to supply the rest of the demand in the retail market. He buys $\theta - q_N$ at the wholesale price p_W and sells it in the retail market at some price p_R^I.

- The prices p_R^I, p_R^E are determined by competition in the retail market under the conditions that E needs to sell all his quantity q_N, and that I needs to supply the remaining demand (there are no more suppliers). We are not involved with the specific structure of the resulting prices.

We can calculate the net profits that I, E make from the wholesale and the retail markets. Let z_I, z_E be the quantities required by the wholesale mechanism to be supplied by I and E respectively. Lets write $z_I = l(q_N) + \Delta_I$ and $z_E = g(q_N) + \Delta_E$, where Δ_I, Δ_E are the extra generation required above the minimum requirements by I, E. Then

In the wholesale market the profits for I are

$$p_W z_I - c_I(z_I) = p_W(l(q_N) + \Delta_I) - c_I(z_I)\,, \tag{2}$$

where $c_I(z_I)$ is the total cost for producing z_I. Similarly, for E we get

$$p_W z_E - c_g(z_I) = p_W(g(q_N) + \Delta_E) - c_g(g(q_N) + \Delta_E)\,, \tag{3}$$

where c_g is the cost of gas.

In the retail market the corresponding profits for I are

$$l(q_N)(p_a - p_W) + (\theta - q_N)(p_R^I - p_W)\,, \tag{4}$$

and for E

$$q_N p_R^E - p_a l(q_N) - p_W g(q_N)\,, \tag{5}$$

since he needs to pay I the lignite part of q_N at price p_a and the gas part at the wholesale price.

Lets compute now the total profits π_I, π_E of I, E respectively. Simple calculations obtain

$$\pi_I = -p_W \Delta_E + p_R^I(\theta - q_N) + p_a l(q_N) - c_I(z_I)\,, \tag{6}$$

and

$$\pi_E = p_W \Delta_E + p_R^E q_N - p_a l(q_N) - c_g(g(q_N) + \Delta_E) \tag{7}$$

$$= \Delta_E(p_W - c_g) + p_R^E q_N - p_a l(q_N) - c_g g(q_N)\,. \tag{8}$$

By observing the first term in (6) we note that that if $\Delta_E > 0$ then the incentive of I in the wholesale market mechanism is to reduce p_W as much as possible (of course while keeping his total cost $c_I(z_I)$ low as well). This is because I is a "net buyer" from the wholesale market. Similarly, by observing the corresponding term in (8) we note that the objective of E in the wholesale market is to maximize $\Delta_E(p_W - c_g)$. And these incentives do not depend on the equilibrium prices of the retail market.

From our detailed analysis of the uniform auction we know that for the expected values of demand in the Greek market, the equilibrium prices are expected to be at the cost of gas, unless I acts strategically and performs price setting using his lignite production. From our previous remark, I will not do that since his incentives are to keep p_W low. But then the price in the pool is not expected to rise above the cost of gas, see Figure 13 (a). This remark is further enhanced by the fact that E must bid less aggressively (even below cost) in order to satisfy the constraint $z_E \geq g(q_N)$ and be chosen to contribute power from his gas facilities by the pool mechanism operator.

The above remarks suggest that there is a strong "decoupling" of the wholesale mandatory pool market and of the retail OTC market. The reason is that the retail prices are not affected by the wholesale prices. This occurs since I buys from the pool and sells to the pool almost equal quantities and hence p_W is not important. Similarly, E revenue in the retail market is independent from the wholesale prices.

Furthermore we can make a prediction for the wholesale prices. For I, the relatively small extra amount he needs to buy from the pool incentivizes low wholesale price, probably near costs. And E expects to make little (if any) of profit by selling to the pool.

A crucial assumption here is that there are no other suppliers competing in the retail market which need to buy all their goods at the wholesale market. If such competition develops in the retail market, say by the new supplier S, then it is not clear if the incentives of I in the wholesale market are to keep wholesale prices low. By raising p_W he must evaluate the negative effect of the term $p_W \Delta_E$ with the positive effects on his profits from the retail market since now his competitor supplier S will operate with higher costs.

APPENDIX

To motivate and prove the structural properties of the uniform auction mentioned in the introduction, we start with a simpler case of the auction where each provider of electricity has a single product to sell, of a given capacity and marginal cost. Then we extend this approach in the actual case of the Greek market where a provider may have different products with different capacities and marginal costs. In this case there is synergy between the bids of the same provider which complicate the analysis, and the results are different than in the case of single products.

The basic idea behind this analysis is that we care to find the equilibrium prices for given parameters of the market. To do that we observe that for given parameters of the auction, if there is a Nash equilibrium with some price p, then there must exist a 'canonical' Nash equilibrium (i.e., an equilibrium that obey some fundamental structural properties), with the same price. Hence we confine our search to such equilibria which are easier to characterize and compute algorithmically. By finding these we characterize completely the prices that are expected to form in the given market.

A Structural results for Nash equilibria

In this section we provide some structural properties of the NEs of the uniform auction. This allows us to confine the search of NEs associated with a given market price to the ones that satisfy these properties and hence are easier to search for.

To help the reader and motivate the results we start with a simpler case where competitors have single energy products to compete in price. Then we analyze the actual case of the Greek market where competitors have access to products of multiple marginal costs.

A.1 A property of the uniform oligopoly auction with providers offering single products

Our model of the market is the following.

Model description

- N providers, provider i has capacity K_i, marginal cost c_i, $c_i < c_j$ for $i < j$.
- Maximum price $P_{max} > c_N$, demand $\theta \le \sum_i K_i$ (if $\theta \ge \sum_i K_i$ then the only equilibrium price is P_{max}).
- Auction: providers submit bids b_i, auctioneer orders bids and fills the demand θ with the capacities of the producers accepting the lower bids first. In case of ties, the provider with lower marginal cost has priority. The resulting auction price p is uniform, i.e., it corresponds to the highest accepted (fully or partially) bid (the 'marginal' bid).

The above auction has multiple equilibria in pure strategies. Another property is that in any such Nash equilibrium (NE) the bids are values from the set $\{c_1, \ldots, c_N, P_{max}\}$.

To show the existence of multiple equilibria we borrow an example from [2]. Let $N = 3$, $c_1 = 0, c_2 = .5, c_3 = 1$, $K_1 = 1, K_2 = 1, K_3 = 0.25$. Furthermore, let $P_{max} = 1.75$ and $\theta = 1.5$. Then it is easily verified that the following equilibria exist in the uniform auction: $\{b_1 = 1, b_2 = .5, b_3 = 1\}$ and $\{b_1 = 0, b_2 = 1.75, b_3 = 1\}$. In a competitive market price should never rise above the cost of the least efficient provider (provider 3) since under truthful bidding the first two providers have lower marginal costs and can fully satisfy the demand. Observe that the first of these equilibria is competitive, whereas the second equilibrium is not (provider 2 sets the price at P_{max} which is above the cost of provider 3). Note further that both equilibria are inefficient in the sense that overall generation costs are not minimized: in both equilibria inefficient dispatch results.

The task of finding all NEs in the general setup is extremely hard to achieve since the search space is vast and there is no straightforward way to reduce it. We like to to reduce the search space by providing some necessary conditions for the existence of Nash equilibria with price p. The idea is that if there is an NE with price p then there must be an other NE, $\bar{\text{NE}}$, with the same price, that has some nice properties which restrict the possible bids. Hence in order to see if there are NEs with price p its enough to restrict our search to the class of $\bar{\text{NE}}$s which is much more restricted since it is characterized by a simple set of necessary conditions. Then to determine if a set of bids that satisfies these sufficient conditions is an NE, we need to check some necessity properties, namely that the given bids maximize for each provider his profit, which is easy to check.

Lets take first for illustrative purpose the case $p = P_{max}$. Consider such a NE. In this NE there must be a non-empty set of bidders that bid P_{max}. Let i be such a bidder with the highest MC (hence he gets lower priority in filling his capacity). *Then we claim that there must also be another $\bar{NE}$ with price P_{max} under which all bidders* except *i bid truthfully their MC*[8]. Observe that any bidder $j \neq i$ which was not truthful in the original NE, must be selling all his capacity anyway at price P_{max}, and hence he is indifferent in bidding truthfully in which case he will get the same result. If he was not selling all his capacity, he could bid his true cost and sell his capacity for profit, contradicting the fact that we were at a NE.

Another way to see this is that any non-truthful bidder $j \neq i$ at the original NE bidding above his MC cannot lose by changing his bid to $b'_j = c_j$ since by lowering his bid he will be asked to provide at least the same amount of capacity as before. Since he was indifferent in doing so in the original NE, he must be selling all his capacity anyway, and he will continue doing so after changing his bid to truthful. Note also that all bidders except bidder i must be filling completely their capacities at the original NE.

The previous arguments suggest that at the new $\bar{\text{NE}}$, assuming that i bids at P_{max}, other bidders don't gain by deviating from being truthful. It remains to show that at P_{max} also i does not gain by deviating from his original bid. It is easy to see that his best possible choice besides P_{max} is to bid his true MC and sell the maximum possible capacity instead of being a price setter and sell less. Lets compare the effect of such an action by i in the original NE and in $\bar{\text{NE}}$. In the first he faces less competition (since bids of competitors may be higher than their MC) than in $\bar{\text{NE}}$ where the competitors bid more competitively (at their MC). Hence the profit of such an action will be more in the original NE than in $\bar{\text{NE}}$. Hence if he is better off by deviation in $\bar{\text{NE}}$ he would also gain by doing so at the original NE, that contradicts the fact that it was an NE.

It is clear that the above arguments lead to a necessary condition that must characterize at least one NE that is associated with price P_{max}. Hence to determine whether P_{max} is the price at *some NE* its enough to consider as candidate NEs only 'truthful' NEs where a single bidder bids P_{max} and all others bid truthfully. There are N such candidate NEs, one for each

[8] If we relax the auction rule that in the case of equal bids, capacity is allocated with priority to small MCs, then a single price setting bidder must exist in such an NE.

i being the highest bidder, and we can easily check each one for 'stability': each bidder cannot improve his profits by deviating assuming the rest of the competitor's bids fixed. Note that this stability condition is the sufficient condition we need to check at each candidate NE. It may be the case that candidate points (sets of bids) satisfy the necessity conditions but not the sufficiency conditions.

We can check that almost the same arguments go though to check if there is a NE for which the auction clearing price p is equal to some c_k, $1 \leq k \leq N$. In this case there must be at least one price setting bid b_i of provider $i < k$ (and hence with $c_i < c_k$), for which $b_i = c_k$. If no such bid existed, then the price must have been set by the bid $b_k = c_k$ of provider k, which gets (in general) partially filled. But then this provider could increase his bid above c_k up to c_{k+1} and make more profit, contradicting the assumption of being at an NE.

Suppose that there are more than a single bidder with bids equal to $c_k = p$. Clearly, from the previous discussion, all these providers have MC $< c_k$. One can easily see that all of them except the one with the highest MC (among them) must be getting full. This is because if i is the price setting provider that bids at c_k and gets partially filled, then any provider with higher MC that bids also c_k and gets nothing would lower his bid (say to his true MC with is below c_k) and make profit.

Now similarly as in the case of $p = P_{max}$, all providers $j < i$ must be selling their complete capacity and hence they are indifferent in bidding their true MC. Regarding bidder i, using the same arguments, since in the original NE he is not gaining by bidding his MC, he certainly does not gain by lowering his bid in this new $\bar{\text{NE}}$. Hence it is indeed an NE.

A sufficient condition for such a candidate NE to be also a true NE is that given the bids of the others, no bidder can improve the profit he is getting. This "stability property' is an easy property to check numerically.

Case of some marginal costs being equal

Observe that our analysis so far led to the conclusion that at any NE there must be a price setting provider which is strategically bidding above his true MC. This conclusion fails to hold when costs are equal in which case truthful bidding might be an NE, i.e., we may get equilibrium prices that are equal to marginal costs. To see this assume that providers have again different MCs, but for some of them these differ by some very small $\epsilon > 0$. Consider the case that $c_{i+1} = c_k + \epsilon$ and i is the price setter bidding c_{i+1}. For example, this corresponds to the case where if all providers bid truthfully, then provider i gets partially filled, and hence he has the incentive to raise his bid to the next higher marginal cost c_{i+1}. He does not raise it above c_{i+1} because then provider $i + 1$ would undercut him.

Then as $\epsilon \to 0$ i will be bidding truthfully since $c_{i+1} \to c_i$. This suggests that we can view the case of equal costs as the limiting situation of cases with different costs as the case analyzed previously. In our analysis of the possible equilibria we need to include the possibility of truth telling NEs and analyze also these for stability. A simple corollary (in the case of some MCs being equal) is that for a given demand, if there is no equilibrium where price is determined by a bidder deviating from his MC, then price is determined by truthful bidding. This corresponds to the limit of non-truthful bidding with non-equal costs when the corresponding cost differences tend to zero.

A.2 Uniform auction with providers selling multiple products with different marginal costs

In this section we model more accurately the Greek electricity market, where competing providers have access to products of different marginal costs. For instance, we anticipate that such products will consist of lignite, NOME mixture (energy generated by mixing in given proportions lignite and gas production), gas, petrol, etc. Each provider will have access to a subset of these products with different capacity constraints. Analyzing this Bertrand oligopoly uniform price auction is more complex than the simple product case since there are synergies within providers in using the different products. One can easily see that the optimal strategy for a provider is not to use bids for each of his sub products as if to maximize the corresponding gains independently. His optimal strategies are different now and must take into account the new structure of his supply. As we will show, we can again obtain some interesting necessary conditions for price-setting NEs where there is tragic deviation from true MCs. Similarly to the previous section, we will show that instead of trying to find all possible NEs, it is enough to confine our search to NEs that have some nice structural properties. These properties allow for efficient algorithms to analyze the auction and determine the equilibrium prices. Our results have been obtained independently of [1] and are consistent with the results obtained for the uniform price auction and the existence of forward contracts.

Our model generalization is rather simple. Assume that a provider consists of a set of 'sub'-providers, each corresponding to a product with a different marginal cost. For simplicity assume that all MCs are different (differ by some $\epsilon > 0$). This is not a crucial assumption, it only simplifies the resolution of 'ties' when different sub-providers bid the same price. Otherwise we need to add some tie-braking rule.

Hence

- A total of N sub-providers, sub-provider i has capacity K_i, marginal cost c_i, $c_i < c_j$ for $i < j$.
- M providers, we abuse notation and depending on the context we denote provider $s \in \{1, \dots, M\}$ as the set of his sub-providers, i.e., $s \subset \{1, \dots, N\}$. Different provider sets are disjoint, and all together cover the set of sub-providers.
- Maximum price $P_{max} > c_N$, demand θ.
- Auction: providers submit bids (c_i, b_i) for all of his sub-providers $i \in s$, auctioneer orders bids and fills the demand θ with the capacities of the producers accepting the lower bids first. In case of ties, the sub-provider with lower marginal cost has priority. The resulting auction price p is uniform, i.e., it corresponds to the highest accepted (fully or partially) bid (the 'marginal' bid).
- Equilibrium: no provider can improve his profits (although sub-providers acting independently could).

We first derive for this more general setup some necessary conditions for an NE, following the logic of the previous section.

In an equilibrium we define the 'price-setter' to be the bidder (sub-provider) with the highest winning bid (the marginal bid), which has the highest marginal cost in case of more than one bidders with equal highest bids (our tie-breaking rule defines that in case of equal prices, bids are considered according to lowest actual MC first). Depending on the context, we may refer as price setter the provider with this price-setting sub-provider .

A.2.1 Necessary conditions for a NE with price p

Lemma 1 Necessary conditions 1: *Suppose there is a NE where the price setter is bidder i with bid $b_i = c_k = p$ (similar if $b_i = P_{max}$), and $i \in s$. Then there is another $\bar{N}E$ where (i) for all providers $s' \neq s$, their sub-providers bid truthfully, (ii) for the price setter s, all sub-providers $j \in s, j \neq i$, with accepted bids ($b_j \leq c_k$) bid truthfully.*

The proof is similar as before. The new outcome must be an NE because one can easily see that every provider $s' \neq s$ gets the same profits as before and if he can improve it by taking some different bidding action, then the same action should have exactly the same effects in the previous NE, which contradicts the fact that it was an NE. Similarly, s chose a bidding strategy that maximized his profits in the first equilibrium. The fact that the rest of the bidders became truthful and these bidders were not price setting, implies that the same strategy is again optimal in the new bidding scenario at the new point.

The next lemma exploits more the structure of the NE. If provider s is the price setter through his sub-provider i, then all his sub-providers with marginal costs higher than i must bid higher than b_i in order not to be included in the allocation. The reason is clear: if any such provider bids below b_i and is included in the allocation, he will generate lower profits per unit of capacity sold than bidder i would if he would get the same capacity (capacity is sold at the same fixed price). Hence he should leave way to bidder i as being more efficient for his provider. Only if i fills then an other sub-provider with higher MC should bid lower in order to sell more capacity. But by definition sub-provider i must be partly full at the equilibrium since he is the price setter.

The following properties hold for the 'truthful' NEs defined above.

Lemma 2 Necessary conditions 2: *In any NE satisfying Lemma 1, all sub-providers $j \in s, j > i$ (i.e., $c_j > c_i$) bid above b_i (abstain).*

Corollary 1 *In any NE with price setter sub-provider $i \in s$, all sub-providers $j \in s' \neq s$ bid truthfully, and all sub-providers $j \in s, j > i$ bid above b_i (do not influence the allocation).*

We can restrict further the NEs to require that the abstaining sub-providers bid exactly at p. This makes it harder for other providers to deviate from their original bids since they will face more competition. Hence if they did not find it profitable to deviate when these bids where higher than p, lowering them at p it makes it even harder.

Lemma 3 Necessary conditions 3: *In any NE satisfying Lemma 2, there is an $\bar{N}E$ where all abstaining sub-providers $j \in s, j > i$ (i.e., $c_j > c_i$) bid exactly at $b_i = p$.*

There is a simple necessary condition for sub-provider i to be price setter with $b_i = c_k$ or $b_i = P_{max}$. Assuming the structure of the NE explained in the previous lemmas, he must be getting partially filled with some positive amount of capacity. Let $K_i(k)$ be the leftover capacity for sub-provider $i \in s$ assuming that he is price setter with bid $b_i = c_k$ for some k at some NE that satisfies the properties of the lemmas above. Then for demand equal to θ,

$$K_i(k) = \theta - \sum_{l \in s' \neq s, l < k} K_l - \sum_{l \in s, l < i} K_l \,. \tag{9}$$

Observe that in the calculation of $K_i(k)$ we assumed that all 'colleagues' of i with higher MCs offered higher bids to abstain from the allocation.

Lemma 4 Necessary conditions 4: *In any truthful NE with price setter sub-provider $i \in s$ bidding $b_i = c_k$, then $0 < \theta - K_i(k) \leq K_i$.*

For $b_i = P_{max}$, we define $K_i(P_{max}) = \theta - \sum_{l \in s' \neq s, l \leq N} K_l - \sum_{l \in s, l < i} K_l$ in a similar way and the lemma holds.

The following simple procedure characterizes the candidate price setting sub-provider of s (assuming that s is indeed price setter with price $p = c_k$ and demand θ):

Corollary 2 *Let $\theta_{-s} = \theta - \sum_{l \in s' \neq s, l < k} K_l$ be the leftover demand after all providers except s have bend served with bids below c_k, assuming they bid truthfully. Let $K_r(s)$, the 'relevant' capacity of s, be the total capacity that he is willing to serve by making profit, when price is c_k (the sum of his capacities with MC below c_k), and restrict attention to the sub-providers of this relevant capacity. Then, if we use θ_{-s} to fill the relevant capacity of s starting with his sub-providers with lowest MC first, the sub-provider which will become partially filled (last to meet this demand) is the price setting sub-provider. If no such partially filled sub-provider exists (because $\theta_{-s} < 0$ or θ_{-s} is larger than $K_r(s)$) then s cannot be the price setting provider under these assumptions for the equilibrium price and demand.*

The previous conditions need to be satisfied (necessary) in at least one NE that is defined with price setter sub-provider i and price p.

Case of some marginal costs being equal

In this case we may also get equilibria where bidders are truthful, see the corresponding discussion in the previous section.

A.2.2 Sufficient conditions for a NE with price p (stability condition)

Finally we like to check if a candidate NE satisfying all the above necessary conditions is indeed an NE, so that we get a set of necessary and sufficient conditions. We need to check that given the bids of the rest of the providers, the profits of the given provider cannot be increased further from the profits he obtains at the candidate point. To check this for any given provider s, we assume the bids of the rest of the providers fixed and start with s selling no capacity at all. This defines a market clearing price p_{-s} (which is equal to P_{max} if demand exceeds available capacity by the rest of the providers). Then s keeps increasing from zero his supplied capacity (say by bidding at price 0 so it is always sold in the market), and observes his real profits as his supplied capacity increases (as his supply increases the market price drops from p_{-s}). He stops when his profit is maximized (as in the case of a monopolist facing a given residual demand). While increasing his supply to the market, he uses the capacity of his sub-providers with the lowest MC first.

Note that this profit function will be discontinuous. Each time the capacity supplied by s crosses a point where the maximum price set by the highest bid of the other providers that defines the clearing price drops (and hence the next to the perviously highest bid of the rest of the providers becomes active while the market price makes a jump to this new bid price), the profit of s will drop (since price is discontinuous and falls) and then keep rising since he will sell more at the given market price.

Observe also that all providers except the price-setting one will have truthful bids. The price setting provider will have the bids of his abstaining sub-providers (see Lemma 3) at the price p.

B Computational procedures

In this section we summarize the computational procedures we follow to analyze the auction based on the theoretical results presented earlier.

A basic problem we like to solve is the following.

Problem 1: Given a price $p \in \{c_1, \ldots, c_N, P_{max}\}$, demand θ, and a given provider s, decide if there is an NE with price p and s being the price setting provider.

The solution is rather simple:

1. Use Corollary 2 to find the candidate sub-provider $i \in s$ that does the price setting. If no such sub-provider exists, then the answer is negative.

2. Else, if $i \in s$ is the candidate sub-provider, construct the candidate NE as follows:

   - All providers besides s bid truthfully,
   - All sub-providers of s with costs less than i bid truthfully, sub-provider i bids c_k, the sub-providers with costs above c_i and below c_k bid also at c_k (they don't get assigned by the tie-breaking rule since they have higher MC than i), and the ones with costs $\geq c_k$ bid truthfully.

3. Check if this set of bids is indeed an NE, i.e., check the stability condition in Section A.2.2.

The above procedure allows us to define for each $p \in \{c_1, \ldots, c_N, P_{max}\}$ and each $s \in \{1, \ldots, M\}$ the indicator function $f^1_{p,s}(\theta)$ that is 1 iff there is a NE with price p and price setting provider s for the given demand parameter θ.

Problem 2: Given a price $p \in \{c_1, \ldots, c_N, P_{max}\}$ and demand θ, decide if there is an NE with price p.

The solution of this problem is simple: decide positively iff for some s Problem 1 has a positive answer.

Equivalently, define for each $p \in \{c_1, \ldots, c_N, P_{max}\}$ the indicator function $f^2_p(\theta)$ that is 1 iff some s, $f^1_{p,s}(\theta) = 1$, and hence $f^2_p(\theta) = \vee_s f^1_{p,s}(\theta)$.

Observe that if we like to plot the prices of the NEs where there is price setting behavior (some sub-provider bids non truthfully) as a function of the demand, we can simply plot the functions $pf^2_p(\theta)$, for each $p \in \{c_1, \ldots, c_N, P_{max}\}$, as a function of θ.

For values of demand θ where $f^2_p(\theta) = 0$ for all $p \in \{c_1, \ldots, c_N, P_{max}\}$, truthful bidding is an NE, and hence the market price is determined by a supply curve corresponding to the actual MCs of the sub-providers.

References

[1] M.-A. de Frutos and N. Fabra. How to allocate forward contracts: the case of electricity markets. *European Economic Review*, 56(3):451–469, April 2012.

[2] N. Fabra, N. von der Fehr, and D. Harbord. Designing electricity auctions. *RAND Journal of Economics*, 37(1):23–46, 2006.