Auctions

C.Courcoubetis, Y.Katsoulakos
G.D. Stamoulis

RAE Seminar: March 1, 2013

Outline

- Introduction and Basic Material
- Simple auctions
- Multi-unit auctions
- NOME

Auctions

1. Introduction

What is an auction ?

- A method for allocating scarce resources based on competition
- Bidding mechanism:
  - The seller (auctioneer) defines the auction rules:
    - how the winner is determined
    - how much he must pay
  - Each buyer chooses a bidding strategy
- Basic distinction: single-unit vs. multi-unit auctions

Examples

- Ancient cases:
  - 500BC: Herodotus mentions about auctions in Babylon
  - Ancient Rome: commercial trading, selling war booty
    - 193 A.D.: auction for the entire empire
- More recent cases: auctions
  - for rare collective items
  - in wholesale markets of fish, flowers, etc.
  - for public contracts
  - in stock market
- Very recent cases: auctions
  - Google Adwords
  - Sales of goods over Internet (e-bay, etc.)
  - Spectrum, bandwidth etc.

An auction is a game

- The auction rules define a game among bidders
- Should use the basic game-theoretic concepts to analyze auctions
  - Define economic objectives of bidders
    - Maximize Net benefit → Maximize (Value – Charge)
  - Try to predict bidding behavior → Look for
    - Dominant strategies
    - Nash equilibria

Dominant strategy

- Best strategy for one player, regardless of how opponents play
- Example: Pay-off matrix of 2-player game
  - Up is the dominant strategy for Row player
  - Knowing this, the Column player will play Left

Column Player

Row Player

	Left	Right
Up	2,3	4,1
Down	1,6	3,5

Nash equilibrium (I)

- Set of strategies from which no player has incentive to deviate
  → Best strategy for individual players,
  if others also maintain their strategies
- Example: The battle of sexes
  - Two pure (non-randomized) Nash equilibria
  - No dominant strategies exist

		Alice	
		Box	Ballet
Bob	Box	2,1	0,0
	Ballet	0,0	1,2

Nash equilibrium (II)

- The best outcomes are not always Nash equilibria
- Example: Prisoner’s dilemma
  - The co-operative outcome
    - is optimal, but …
    - is not an equilibrium

Prisoner 2

Prisoner 1

	Coop.	Def.
Cooperate	3,3	0,4
Defect	4,0	1,1

Alternatives to auctions

- Negotiate
- Set fixed prices (take it or leave it)

Assume independent buyer valuations, unknown to seller $V = (V_1, \ldots, V_n)$

  - distributions iid, uniform on [0,1], **known to seller**
  - problem: find price that maximizes seller's net profit

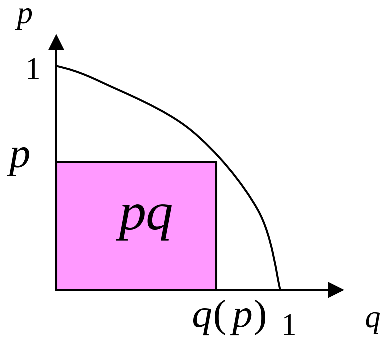
$$\Pr\{\max V_i \geq p\} = 1 - \Pr\{V_1 < p, \ldots, V_n < p\} = 1 - p^n$$

Expected quantity sold at price *p* : $q(p) = 1 - p^n$ = Demand function

Expected revenue: $R(p) = q(p)p = p(1 - p^n)$

$R(q) = qp(q)$

Optimal price: $p^*(n) = \sqrt[n]{\dfrac{1}{n+1}}$ $R = p^*(n)\dfrac{n}{n+1}$



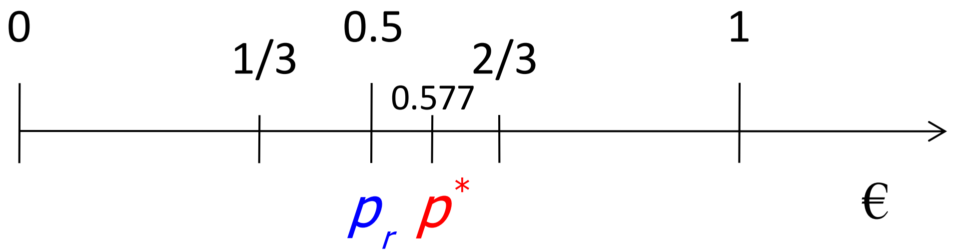
An example of mechanisms

$N = 2$ buyers, valuation iid [0,1]

1. Optimal price $p^* = \sqrt{1/3}$

Expected revenue = 0.3849

Efficiency = 0.52578



2. Auction (first/second price)

Expected revenue = 0.3333

Efficiency = 2/3 = 0.6666

3. Optimal auction (second price)

Reserve price = $p_r = 1/2$

Expected revenue = 0.4167

Efficiency = 0.5833

Note: Trade-off between <u>allocative efficiency</u> and <u>revenue</u>
Optimal auctions outperform simple market mechanisms

Efficiency = Social Welfare

Markets are not enough

- In order to obtain efficient allocations we need
  - Large numbers of bidders
  - No externalities (allocative, informational)
- Not the case in general, need **special market process**
  - Pay more attention to details of market mechanism
  - Take into account private information differences
  - Balance distortions due to market power

Why auctions ?

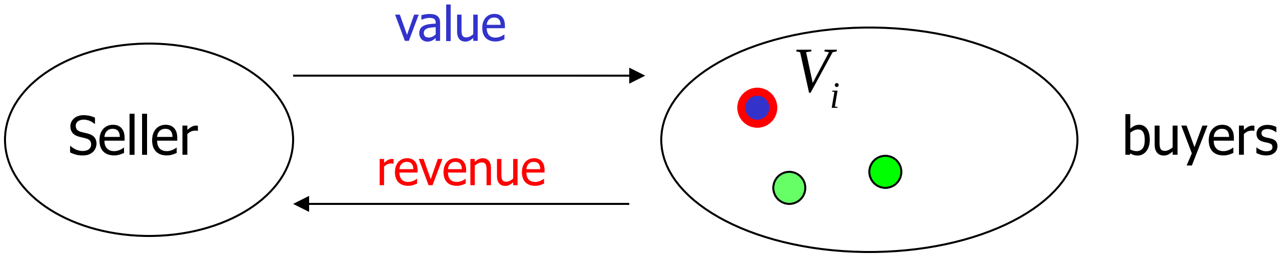
- Useful in the context of incomplete information
  - when selling a commodity of undetermined value
    - value depends on buyer, or actual value found ex-post
  - when no information is available about value distribution of buyers, i.e., demand
  - Bidders also learn from the auction
- Auctions offer transparency, speed of sale and reveal information about buyer’s valuation and demand
  - Allocation of good(s) and price(s) are determined by the market response,
  - … thus preventing dishonest dealing between seller-buyer

Other benefits from auctions

- Information aggregation and revelation
  - Data collected in auctions for liquidity by central banks drives monetary policy
- Foster competition
  - In privatization environments
  - Create new market architectures
  - Mimic stock exchanges, but take into account special properties of traded goods
  - In cases private monopoly is not avoidable, limit time period of rights

Auctions as allocation mechanisms

- An auction is a market mechanism that
  - allocates resources (goods) to buyers
    - generates value for the consumers
  - generates revenue for the seller
    - generates revenue for the producer
  - can also serve as an internal allocation mechanism



Performance measures

- When choosing an auction design, a variety of assessment criteria and measures may be used:
    - social efficiency

      → allocation that maximizes the total value to buyers
    - revenue → seller's profit
    - bidders' profits
    - time to run the auction
    - implementation complexity
    - simplicity of rules
    - simplicity of strategies
    - susceptibility to collusion

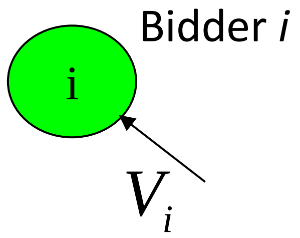
Bidder and seller characteristics

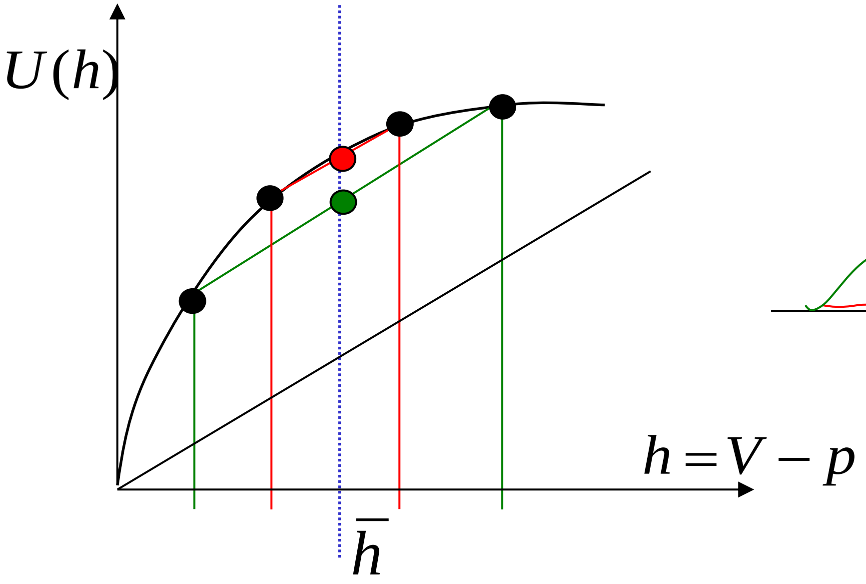
- Valuation
  - private values
  - common values
  - correlation
- Risk assessment
  - risk neutral
  - risk averse
- Symmetry
  - symmetric
  - asymmetric

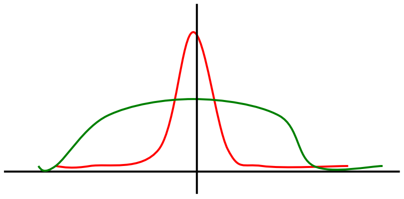
$$U_i = V_i$$

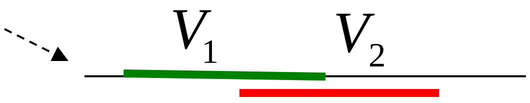
$$U_i = V, V_i = V + x_i$$

$$U_i = aV_i + b\sum_{j \neq i} V_j$$









Key economic issues that influence the outcome

- Informational externalities (common values, prior info,…)
  - open bidding formats (but collusion incentives)
- Allocative externalities (valuation depends on what my adversaries get)
  - Monopolist valuation if he obtains all licenses
  - Buying a non-valuable patent so that it does not fall to competitors
- Homo/heterogeneity, unit/multiunit, complementarity/substitutability among goods
- Budget constraints
- Risk aversion
- Bidder symmetry

Other key design issues

- Identify what various bidders want to achieve, what will influence their bidding behavior
- Define what objects to be auctioned, and rules
  - Demand and supply depend on many factors
  - Structuring of traded goods and bids should represent bidder’s preferences
  - Examples: European UMTS auctions: unit demand, heterogeneous; German-Austrian: identical blocks, multi-unit, complementarities

Externalities and market influence (I)

- Bidders care **where** the rest of the items are allocated
  - It influences future market competition, revenues
  - A bidder may fear that it will incur future losses if another strong competitor wins (due to future interaction)
  - Willingness to pay depends on what will happen if he does not get the object (hence not a socially efficient consideration)
  - Overbidding and inefficiency

Externalities and market influence (II)

- Example: Licenses: A,B, firms: 1,2, can buy 1-2 licenses.
- If each gets one license due to eventual competition future profits = 0.
- If firm i gets both, it earns monopoly profit π_i, $\pi_1 > \pi_2$.
  - Auction outcome: firm 1 gets both licenses, bad for consumers!
  - Alternative: each firm can buy at most 1 license. Then auction revenue = 0!
  - Tension between short run and long run social benefit
- One must control flexibility in order to control eventual market power

Auctions

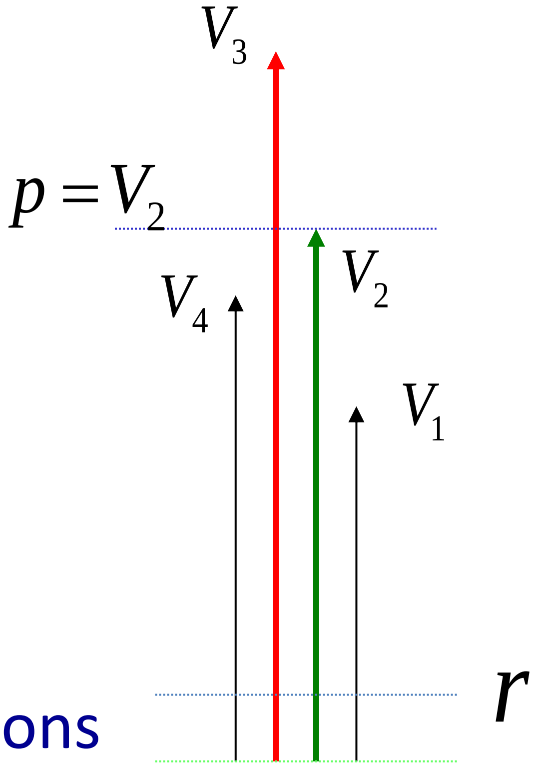
2. Single-unit auctions

Types of single-unit auctions

- Basic types:
  - English
  - Dutch
  - First price sealed bid
  - Vickrey (second price sealed bid)
- Rare types:
  - *k*-th price, all-pay,...

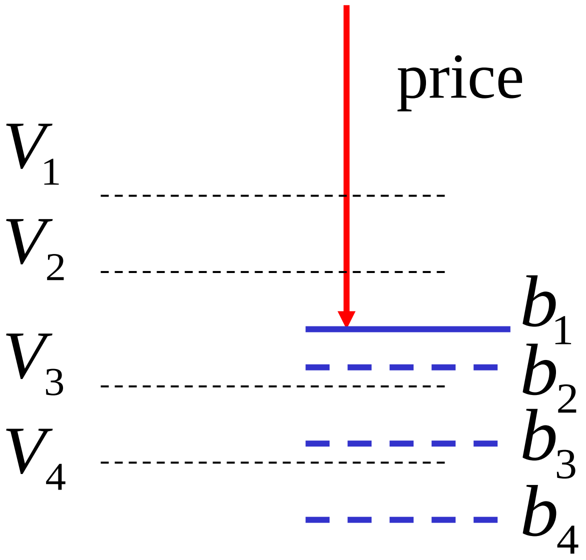
English auction

- Ascending bid, open-outcry
  - other formats: as in Japan, ...
- Top bid wins
  - Item is sold at least at the reserve price
- Dominant strategy for a bidder
  - bid a small amount more than the previous high bid until bidder’s valuation is reached, then stop
- Auctioneer has great influence
- Most emotional and competitive of auctions
- Much information regarding demand is revealed



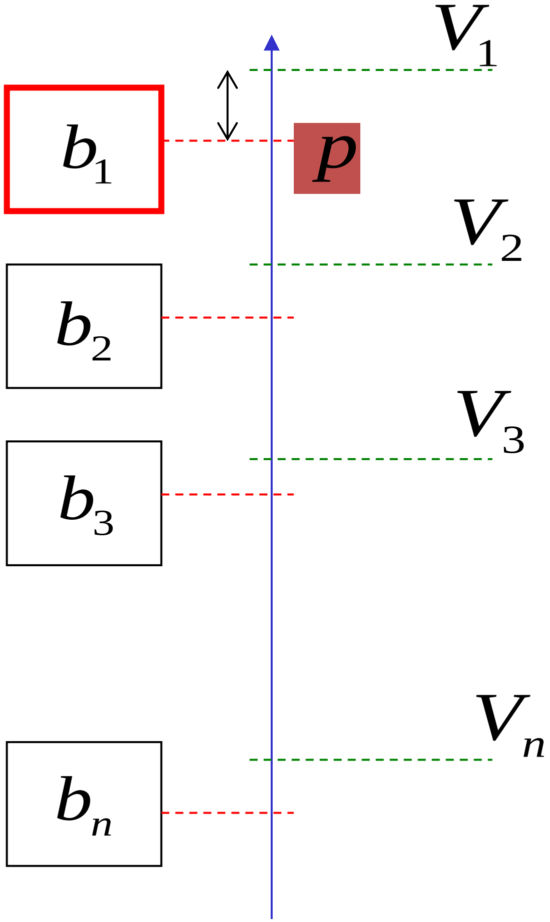
Dutch auction

- Descending price, often by "dutch clock"
- Open-outcry, first price wins
  - Winner → bidder that first shouts "mine"
- Auctioneer usually has no influence
- Little information on demand is revealed



First price sealed-bid auction

- First price (top bid) wins
- Sealed: each bidder is ignorant of other bids
- Usually one bid allowed per bidder
- Performed in two phases
  1. bidding
  2. Resolution, i.e. winner determination
- Bidder's strategy: shade bids
  - to generate a profit due to pay-your-bid
  - to avoid winner's curse (for common value)
- Little information revealed about demand

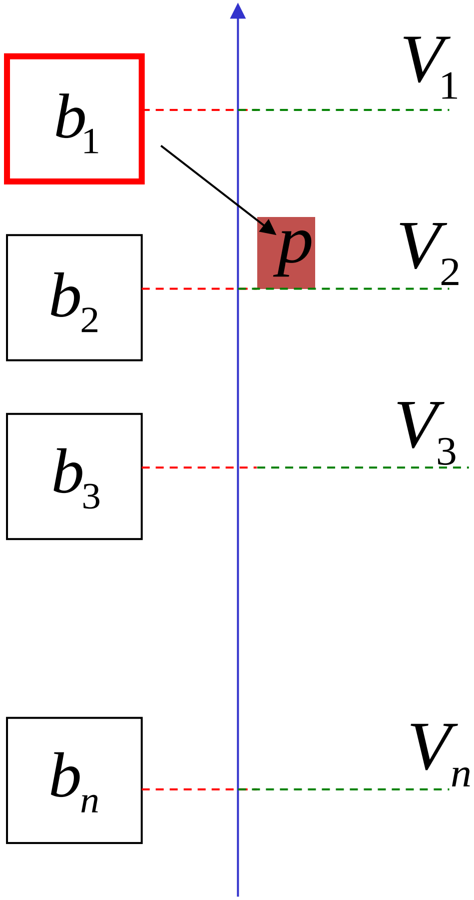


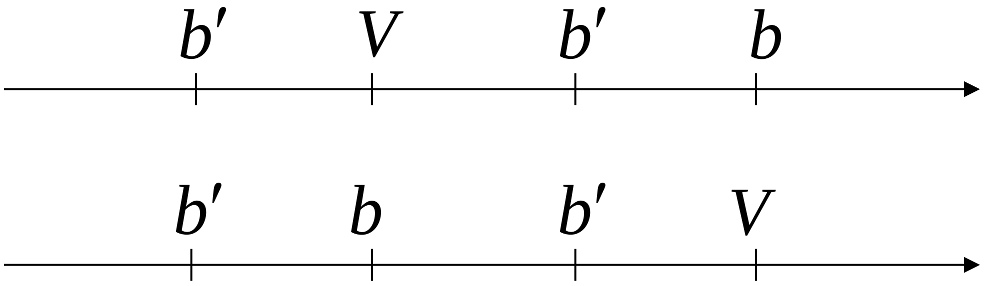
Vickrey (2nd price sealed-bid) auction

- Sealed
- The item is awarded to the highest bidder at a price equal to the second highest bid
- Dominant strategy: submit a bid equal to true valuation

  → incentive compatibility

  guarantees efficiency
  - less fear of winner’s curse (for common value)





Basic results

- Dutch and First price sealed bid auctions:
  - strategically equivalent, i.e. one-to-one mapping of strategies → equivalent payoffs
- English and Vickrey auctions:
  - equivalent in outcome, under private values,
  - … but not strategically equivalent
    - only in the English auction bidders can respond to rivals
- All four auctions are equivalent in terms of expected revenue, if SIPV model applies ➔

The SIPV model

- Assume:
  - a single indivisible object
  - private values
  - all bidders are indistinguishable (symmetry)
  - valuations are independent and identically distributed continuous random variables
  - bidders and seller are risk neutral
- Revenue Equivalence Theorem:
  - *all auctions that: award the item to the highest bidder and lead to the same bidder participation are payoff equivalent*

Procurement auctions

- So far, overviewed auctions for selling a good
- Similar auction types for buying a good, yet the direction of price progress is reversed
  - E.g., in the English auction, a new valid bid should be below the standing bid
- Similar results apply on strategies, equivalence properties etc.

Roadmap

- So far, for single-unit auctions:
  - Defined the basic types
  - Stated and applied Revenue Equivalence Theorem

- Issues to follow:
  - Variations: asymmetric cases, risk averse bidders, common value auctions, participation fees
  - Collusion
  - Comparing open vs closed auctions

Asymmetries

- Revenue equivalence no longer applies
- Second-price auctions: optimal strategy not affected, efficient, no longer revenue maximizing
- First-price auctions: optimal strategy crucially depends on beliefs, hard to calculate equilibria, no longer efficient or revenue maximizing
- Revenue comparison: can go both ways

Risk averse bidders

- In a first-price auction, risk averse bidders shade less their bids, thus resulting in
  - higher average prices and seller revenue
  - lower expected bidder profit
  - They have no control of their payment in a second-price auction
- With risk averse bidders, the seller prefers the first price (SB, Dutch) to second price auctions (English, Vickrey)
- Revenue equivalence no longer applies

Common value auctions

- Value of bidder is not fixed before the auction
  - True value of item is not known ex-ante, although defined
  - Value to bidder i depends on other bidder's values
  - Examples: sealed box with coins, oil-lease

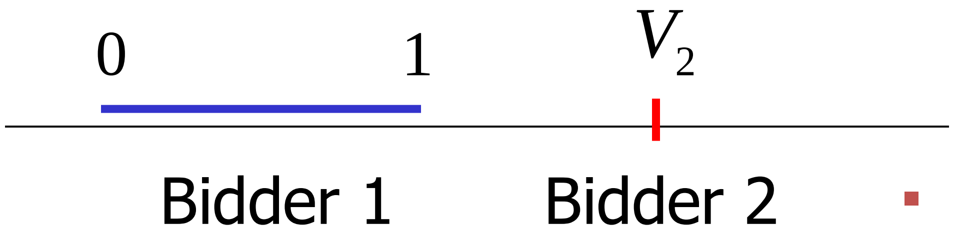
- Complex strategies, no general results
- Winner's curse: the winner discovers that he overestimated the value of the item
- Strategic approach: shade the bid to account for the adverse selection bias

Bidding in common value auctions

- In an ascending price auction: take into account the information revealed by others, avoid overbidding
- Gradual release of information, possibility to react
- When positive correlation between bidding signal of i and expected value of j, then ascending price auction generates higher revenue than second price auction
- Efficiency if highest signal implies highest value

Asymmetric cases

- Different distributions for bidders' valuations
- Revenue equivalence does not apply
- First price auctions are not socially optimal
- Public authorities should use second price auctions for efficiency purposes
  - otherwise, possibility for inefficiency



$$\Pr\{b_1 > b_2\} > 0$$

- e.g., if bidder 1 is truthful, then b_2= min{1,1/2*V_2}

Participation fees

- Seller imposes a participation fee *c*

- Buyer participates only if his expected payoff is larger than *c*

- Fiercer competition → higher expected seller revenue

- Positive probability of non-participation
  → not socially optimal

- **Optimal auction design**
  - Complex winner determination and second price mechanisms

Collusion (I)

- Bidders form collusion rings, within which they agree to get the item at a lower price:
  1. They select their designated winner
→ the one with the highest valuation
  2. Others promise to follow a specific strategy
→ abstain from bidding

- Which auctions are more vulnerable to collusion than others ?
  - Enforcement issue: incentives for non-winners to keep their promise

Collusion (II)

- First price sealed bid and Dutch auctions:
  not self-enforcing! no possibility for punishment
  - In FP: winner places bid = $p_{\min} + \varepsilon,$ other bidders may abstain or break the ring by bidding slightly higher
  - In Dutch: one of the others may shout "mine" and win!
- English and Second price auctions: self-enforcing!
  - In English: if one of the others bids higher than promised, then the winner may overbid again
  - In SP: winner's bid = valuation, others' bid =0

Open vs Sealed-bid Auctions (I)

- Advantages of open auctions:
  - Revelation of information about demand
    - Helpful in the development of strategies
    - Limits the uncertainty about the good’s actual value
  - Prices reflect competition
  - Bidders do not reveal their maximum willingness-to-pay

Open vs Sealed-bid Auctions (II)

- Disadvantages of open auctions
  - – Possibility for low revenues in cases of low competition → zero-price equilibria
    - May apply to sealed-bid auctions too, to a lesser extent
  - Collusion is self enforcing
    - Does not apply to sealed-bid auctions
  - Less competitive players may decide not to participate at all, thus limiting competition
  - Higher complexity in terms of:
    - Implementation and running overhead
    - Development of strategies

Auctions

3. Multi-unit auctions

Types of multi-unit auctions

- Multiple items are for sale
- Classification of multi-unit auctions:
  - Are all items the same ?
    - Yes: homogeneous, No: heterogeneous
  - Are all items put for auction at the same time ?
    - Yes: simultaneous, No: sequential
  - Do prices evolve before being finalized ?
    - Yes: progressive, No: one-time sealed bids
  - For heterogeneous: are bids for combinations allowed ?
    - Yes: combinatorial, No: individual
  - For homogeneous: are all units charged at the same price ?
    - Yes: uniform price, No: discriminatory

The simple case (unit demand)

- *k* units (**identical items**) are for sale
- Each bidder can only bid for one unit
- First price auction: *k* highest bids win, each winner pays his bid
- Second price auction: *k* highest bids win, each winner pays the value of the *k*+1 highest bid (the highest loosing bid)
- Revenue Equivalence applies

Multi-unit demand

- When bids for multiple units are allowed:
  - Revenue Equivalence does not apply
  - key issue: complements and substitutes
- If objects are **heterogeneous** a simultaneous ascending-price auction is efficient

- Simple case: homogeneous items
  - **Interesting sub-case: items are substitutes**
  - Value of *k*-th item for a user is less than his value of (*k*-1)-th
- Use Generalized Vickrey (next)

Generalized Vickrey Auction

- Sealed multi-unit auction

- Winners are charged according to the social opportunity cost their presence entails
  - Extension of the single-item Vickrey auction

- Incentive compatibility still holds: truthful bidding is the dominant bidding strategy
  - Unlike "pay-your-bid" auctions, bid shading is not beneficial

- The GVA outcome can also be attained by an open format: "Ascending Clock with Clinching"
  - Under specific information-revelation to the bidders

Example of a GVA

Bids	1st unit	2nd unit	3rd unit
A:	50	40	*37*
B:	*35*	32	30
C:	30	25	10
D:	42	*38*	31

- Three units are auctioned to four bidders
  - A wins two units and is charged 38+35=73
  - D wins one unit and is charged 37

Problems with uniform price auctions

- Demand reduction: large bidders may prefer to withhold part of their demand, in order to depress prices
  - arises with homogeneous items (and in general substitutes), particularly for uniform pricing
  - Results in lower revenues for the seller and possibly an inefficient outcome (for asymmetric players)
  - Can be alleviated by:
    - offering discounts to large buyers, or
    - discriminatory pricing, although differential bid shading arises then!

Example of demand reduction

- Assume: uniform price, which equals highest loosing bid
  - Bidders #2 and #3 demand a single unit → truthful
  - Efficient outcome:
    - AB → Bidder#1, social welfare = 120
    - attained if #1 is truthful → price=40, profit (net benefit) of #1 = 40
  - Most profitable outcome for Bidder#1
    - bid 70 for one unit and 0 for the other
    - outcome: A →Bidder#1, B → Bidder#2
    - price =20, profit of #1 =50, social welfare = 110

Bidder	A or B	AB
#1	70	120
#2	40	40
#3	20	20

Sequential auctions

- Items are auctioned sequentially one-by-one
  - Simple solution for both homogeneous and heterogeneous

- Problems:
  - Identical items can be sold at very different prices
    - Declining-price anomaly often arises
  - Predatory bidding to drive prices high and squeeze rivals
  - Complicated strategies
  - Inefficient aggregation of complements

- What if all items are auctioned simultaneously yet individually ? →

Examples of sequential auctions

1. 3 symmetrical bidders, 2 items, Vickrey pricing
    - 1st round: bid less than ordinarily, benefit from option to get the item in 2nd round
    - 2nd round: reduced competition
    - 3rd round: last round, fiercer competition

2. Real case → RCA-satellite auction, 1981
    - 7 licenses for cable television broadcast
    - Highest \$14.4 M (1st), lowest \$10.7 M (6th)
    - FCC nullified the auction as "unjustly discriminatory" → enforced a uniform price

Complementarities and exposure

- More general case of items, not necessarily homogeneous
- Complementarities create significant difficulties when bids are allowed only on individual items ("individual bidding")
- Individual bids cannot express the magnitude of complementarities and create inefficiencies
- The theoretically correct way to deal with complementarities is to allow bids on bundles → "combinatorial bids"
- Not allowing combinatorial bids lead to the "exposure problem"

Individual bidding is not efficient

- In general, there do not exist prices so that
  - each item is demanded by one bidder only
  - and the outcome is efficient, taking valuations of combinations into account
- The problem can only arise if items are complements
- Example:

Bidder	A or B	AB
#1	a	2a+c
#2	a+d	2a+d

synergy
between
A and B

  - If d<c: most efficient outcome is Bidder#1→AB
  - If c/2<d<c: there do not exist equilibrium prices p_A and p_B such that both items are demanded only by Bidder#1

The exposure problem with individual bidding

- Exposure: Due to an unsuccessful attempt to win a set (combination) of items, a bidder may:
  - win only a subset of these items
  - and pay for these more than his respective valuation

- Arises in cases of complementary items
- Results in an inefficient outcome and/or bidder losses

The exposure problem (II)

- Example of exposure: two driveways A, B
  - Bidder 1 has 2 cars, needs both
  - Bidder 2 has 1 car

Bidder	A or B	AB
#1	0	100
#2	75	75

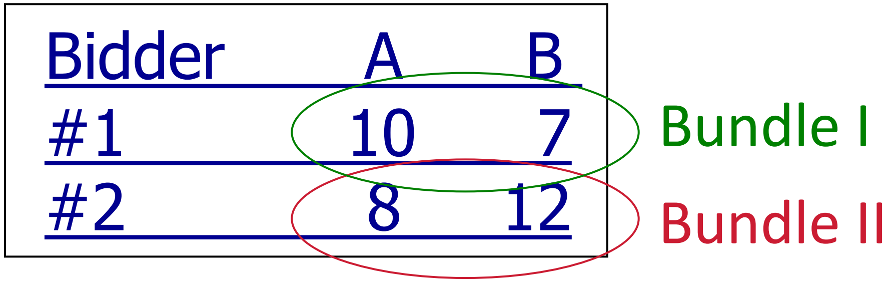
  - Two sequential ascending auctions:
    Assume full information→ Bidder#1 does not participate!
    - Bidder#2 can raise the price in the A auction more than 26
    - Then, Bidder#1 for sure cannot win in both auctions anymore
  - With incomplete info: exposure can arise
    - if Bidder#1 wins the A auction and then loses the B auction, due to underestimating the valuation of Bidder#2

Allowing Combinatorial Bids

- Winning subset of bids = subset of non-overlapping bids resulting in maximum total revenue
- Advantages
  - increased efficiency
  - increased revenue, particularly with reserve prices
  - elimination of exposure problem
- Disadvantages
  - complexity of determining the winners can be very high
    - exponential number of combinations
    - considerably larger number of possible allocations
    - solution: limit permissible combinations
  - **threshold** and **free-rider** problems

Efficiency and revenue maximization

- Assume that all items need to be sold (no reserve price)
- In a single object scenario, efficiency and revenue are maximized together
- In the multiunit case this is not the case!
- The seller gains more by **selling items in bundles**, but then the items are not allocated optimally individually
- Example:



Bidder	A	B
#1	10	7
#2	8	12

  - Using a second price auction for bundles, we get 17$
    - but item A is erroneously allocated to Bidder#2
  - Using two second price auctions for individual items we get only 15$

FCC Simultaneous Ascending Auctions

- Multiple spectrum licenses put in auction:
  - simultaneously, in discrete rounds
  - the bids:
    - are sealed, then announced at the end of the round
    - are placed per individual license
    - should exceed the corresponding highest previous bid
  - rules for maintaining bidding-activity force bidders to participate, otherwise their eligibility (in terms of their target licenses) is reduced
  - termination rule: one round with no bids at all
  - “pay-your-bid” pricing
- Considered a simple yet successful solution

Example of an FCC Auction

	Round →	1	2	3	4	5	6
Licenses	Bidders and Bids ↓						
A	West	10	**25**			**45**	
	North	**15**		30			
	South	5	20	**35**			
B	West	20	**35**		**50**		Termination
	North	15	30	**45**			
	South	**25**		40			
C	West	15	**25**				
	North	10				**40**	
	South	**20**			**30**		
Future Eligibility	West	3	3	3	2	2	
	North	3	2	2	1	1	
	South	3	3	2	2	2	

Activity rule: eligibility(t+1) = activity(t)

activity(t) ≤ eligibility(t)

Final allocation: A → West (for 45),
B → West (for 50),
C → North (for 40)

Auctions

4. Issues about NOME auctions

NOME brief reminder (I)

- NOME: Nouvelle Organisation du Marché Electrique
  - incumbent offers mandatorily access to inexpensive lignite power to third parties.
  - to be injected in the domestic market
    - on the basis of bilateral contracts (OTC – Over The Counter)

NOME brief reminder (II)

- Basic regulatory parameters/tools:
  - access price (p^a)
  - maximum percentage of incumbent’s NOME production to which third-parties can have access (α),
  - highest permissible percentage of NOME capacity per unit of energy independently produced or imported by a third party (φ)
    - Can be enforced at different timescales, allowing different flexibility levels
- Incentive compatibility: A third-party will participate to NOME only if it will obtain higher profits.
- Thus, a third-party may decide:
  - not to participate at all
  - not to purchase the entire NOME capacity it is actually entitled to

Alternative goals of NOME auction

- Assume parameters α and φ are pre-specified

1. Price discovery and NOME capacity allocation with a given reservation price

2. Rule for NOME capacity allocation under a given cost-based access price
    - “Rationing rule”
    - Proportional, priority based etc.

Important Issues on NOME auction

- Reservation price should be representative of cost, due to possibility for limited competition
  - Also applies to the case of rationing

- Prices may go high, and restrict NOME advantage, if competition is severe
  - Possibly specify a price cap, and employ capacity rationing if reached

- Each auctioned “good” specified as a pair [NOME capacity, timescale]
  - Should all contracts/units be auctioned simultaneously?
  - How about contracts covering multiple overlapping timescales;

- Can the parameter φ or (even α) be internalized in the auction design?

Auctions

4. Conclusion

Conclusion

- The design of a successful auction is an art
- There is no rule prescribing the best auction in each different case!
- There exist several auction design techniques that can prove useful in particular practical cases
- Key issues
  - Understand demand, asymmetry of players, preferences
  - Set the appropriate goals (efficiency, revenue, competition enhancement, …)
  - Define items to auction (size, symmetry, parameters)
  - Decide on number of rounds, open/closed, etc.
  - Set appropriate reserve prices
  - Add special rules to address specific aspects
  - Perform simulation and analysis

Selected References

- L.Ausubel and L.Cramton, “Demand reduction and inefficiency in multi-unit auctions”, Technical Report, Univ. of Maryland, March 1998, available at www.ausubel.umd.edu
- P.Klemperer, “Auction theory: a Guide to the literature”, in Journal of Economic Surveys 1999.
- Peter Cramton, “Money Out of Thin Air: The Nationwide Narrowband PCS Auction”, Journal of Economics and Management Strategy, 6:3, 431-495.
- P.Milgrom, “Putting auction theory to work: the simultaneous ascending auction”, Technical Report, Stanford Univ., May 1998.
- R.Tenorio, “Some evidence on strategic quantity reduction in multiple-unit auctions”, Economics Letters, v.55, pp. 209-213, March 1997.
- P.Jehiel and B.Moldovanu, “An economic perspective on auctions”, June 2002.